# The Spin Wave Gap and Switching Field in Thin Films with In-Plane Anisotropy

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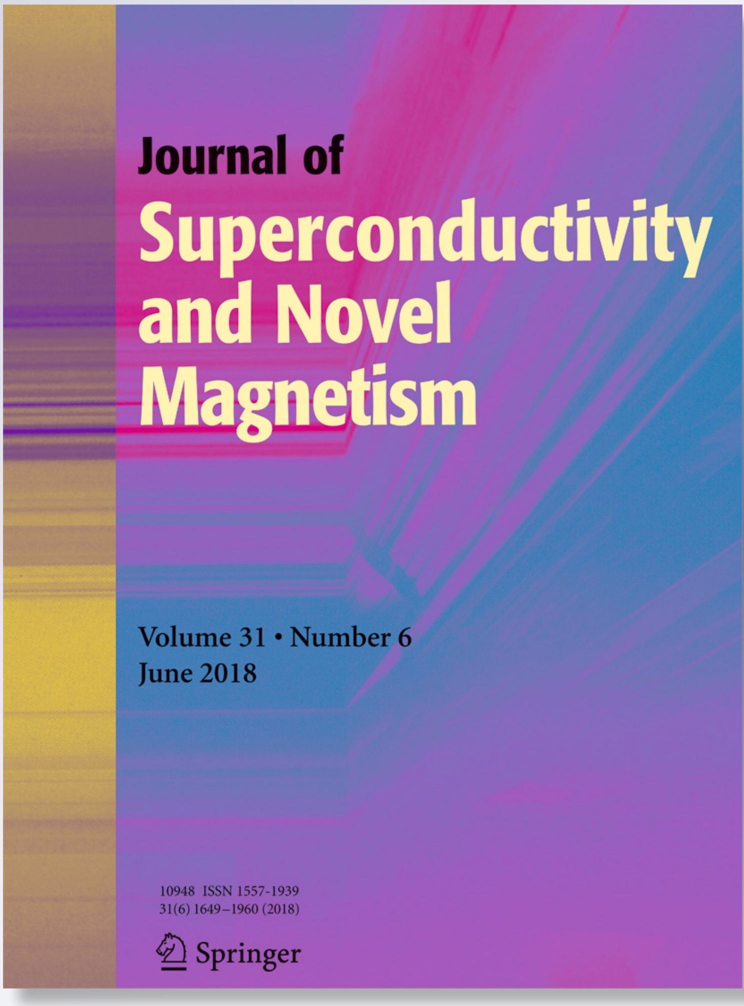
**Journal of Superconductivity and Novel Magnetism**
Incorporating Novel Magnetism

ISSN 1557-1939
Volume 31
Number 6

J Supercond Nov Magn (2018)
31:1779-1783
DOI 10.1007/s10948-017-4395-8




Springer

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https://doi.org/10.1007/s10948-017-4395-8

ORIGINAL PAPER

# The Spin Wave Gap and Switching Field in Thin Films with In-Plane Anisotropy

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Received: 24 August 2017 / Accepted: 19 October 2017 / Published online: 30 October 2017


**Abstract** In this paper, we have calculated the spin wave gap and the angular dependence of magnetization reversal in a single-layer thin magnetic film that includes the strong perpendicular magnetic anisotropy and in-plane anisotropy. The film is assumed to be under the influence of the out-of-plane direction of the applied magnetic field at zero temperature. Using the quantum model, it is shown that the calculated equations present a nonzero spin wave gap at zero magnetic field which is strongly affected by anisotropies. The effects of the in-plane anisotropy and the role of the applied field were examined. We also discussed a simple theoretical model for the angular variation of switching field by using a quasi-classical argument. We used some constants in connection with experimental data which are reported for chromium telluride thin films grown by molecular beam epitaxy.

**Keywords** Magnetic anisotropy · Spin wave gap · Switching field

## 1 Introduction

There has been a growing interest in thin films and two-dimensional magnetic materials connected with the magnetic anisotropy which is closely related to the utilization of direction of magnetization for new technologies such as spin-based devices. Under the influence of film thickness, temperature, and external magnetic field, these materials exhibit complicated phenomena [1–4], arising from film properties such as structure, magnetic, and transport, which in turn are determined by growth conditions [5]. In this context, to precisely measure the magnetic anisotropy field and the constant of anisotropy of magnetic materials has long been an important aspect in magnetic research.

Magnetic anisotropy plays a key role and leads to an energy gap in the spin wave spectrum. Such a gap behaves as an energy barrier to the excitation of long wavelength spin waves, therefore allowing for a finite-order parameter at finite temperatures. The spin wave gaps in ultrathin films under the combined influences of the applied magnetic field and with different kinds of magnetic anisotropies were extensively studied [6–14].

Quite recently, Pramanik et al. [15] have observed strong perpendicular magnetic anisotropy in chromium telluride ($Cr_2Te_3$) thin films grown by molecular beam epitaxy. They have found that the thin films show uniaxial anisotropy along the *c*-axis with a rather strong second-order uniaxial anisotropy term, along with the first-order term. They have also reported the magnetization reversal mechanism which is the angular dependence of switching field from a 4-nm-thick film observed from the magnetoresistance measurement. The stabilization mechanism of magnetization is closely related to the in-plane anisotropy [16] and therefore, this raises the important question concerning the influence of the in-plane anisotropy contribution and the direction of the applied field, which requires control of the spin wave gap. The magnetic anisotropy contribution and the direction of the applied field strongly influence the spin wave gap of an ultrathin film with small thickness.

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The purpose of this paper is to present a simple quantum model which gives the spin wave gap as a function of the applied magnetic field which is perpendicular to the film plane at zero temperature. Besides, it is of interest to look at the angular variation of the switching field which may be calculated from a quasi-classical argument.

## 2 Theoretical Model

We first consider a film in the $(x, y)$ plane and assume that the total Hamiltonian can be expressed generally as follows:

$$\mathcal{H} = \mathcal{H}_a + \mathcal{H}_Z \tag{1}$$

where the Hamiltonian $H$ contains an anisotropy term and the Zeeman term, respectively. It is assumed that all magnetic atoms have the same spin quantum numbers. The first term of (1) represents the magnetic anisotropy energy which has the following form:

$$\mathcal{H}_a = \sum\nolimits_n \left[-K_1(n)\, S_z^2(n) - K_2(n)\, S_z^4(n) + K_{\text{in}}(n)\, S_y^2(n)\right] \tag{2}$$

where $K_1$ and $K_2$ are the first- and second-order perpendicular uniaxial anisotropy constants, respectively, $K_{\text{in}}$ is the constant and describes uniaxial in-plane anisotropy which breaks the fourfold symmetry so that the film is a cubic material, with the $z$-axis being taken as the surface normal. All of the anisotropy constants depend on the film thickness $L$. The sign convention contained in (2) implies that positive (negative) values of anisotropy constants favor the magnetization lying perpendicular to the film plane (in the plane). The second term of (1) implies the Zeeman energy of the spins in a magnetic field of magnitude $H$ which throughout we suppose is directed along the $z$-axis. Equation (1) can be defined with respect to some local axes $(\lambda, \mu)$. We use the transformation between the local axis and the axis defined by the anisotropy [17]:

$$\begin{aligned}\mathcal{H} = \sum\nolimits_n \Big\{&-K_1 S_z^2(n) - K_2 S_z^4(n) + K_{\text{in}}\Big[S_y^2(n)\\ &+\left(S_x^2(n) - S_y^2(n)\right)\sin^2(\theta)\\ &-S_x(n)\, S_y(n) \sin(2\theta)\Big] - g\,\mu_B H\, S_z(n)\Big\}\end{aligned} \tag{3}$$

where $g$ is the gyromagnetic ratio and $\mu_B$ is the Bohr magneton. We use the Holstein–Primakoff transformation [18] to examine the spin wave gap at zero temperature. According to our choice of coordinate axes, the components of a spin vector $S(n)$ are represented in terms of boson creation and annihilation operators $a_n^\dagger$ and $a_n$ by the following expressions:

$$S_x(n) = \frac{\sqrt{2S}}{2}\left(a_n + a_n^\dagger\right),$$
$$S_y(n) = \frac{\sqrt{2S}}{2i}\left(a_n - a_n^\dagger\right), \quad S_z(n) = S - a_n^\dagger a_n \tag{4}$$

The spin operators are $S_x$, $S_y$, and $S_z$, and they are defined by their commutation relations $\left[S_x, \ S_y\right] = i\,\hbar\, S_z$ and so on [19], and at low temperature, $S_z \approx S$. More carefully, substituting (4) in (3), one can find a different Hamiltonian which is diagonalized by the introduction of spin operator $S_\alpha$ where $\alpha = x, y$ that is chosen to satisfy the relation $[S_\alpha, H] = \omega_\alpha S_\alpha$ [20]. The eigenfrequencies are the roots of

$$\det\begin{vmatrix}\omega_o - A_o & -B_o\\ B_o & \omega_o + A_o\end{vmatrix} = 0$$

or

$$\omega_o = \left(A_o^2 - B_o^2\right)^{1/2} \tag{5}$$

For general angles $\theta$ between the $z$-axis and the vector $S_z$, the secular equation is given by (5) with

$$A_o = g\,\mu_B\left(H + \varepsilon_o + \varepsilon_1\cos^2(\theta)\right)$$

and

$$B_o = g\,\mu_B\left(H + \varepsilon_o + \varepsilon_1\sin^2(\theta)\right)$$

where $\varepsilon_o = 2\,H_1 + 4\,H_2$ and $\varepsilon_1 = 2\,H_{\text{in}}$. Here, it is useful to introduce the anisotropic fields $K_1 = g\,\mu_B\,H_1$, $K_2 = g\,\mu_B\,H_2$, and $K_{\text{in}} = g\,\mu_B\,H_{\text{in}}$. For $\theta = 0$ and 90°, we find $(S = 1)$ the following expression:

$$\omega = g\,\mu_B\,[(H + \varepsilon_o)\,(H + \varepsilon_o + \varepsilon_1)]^{1/2} \tag{6}$$

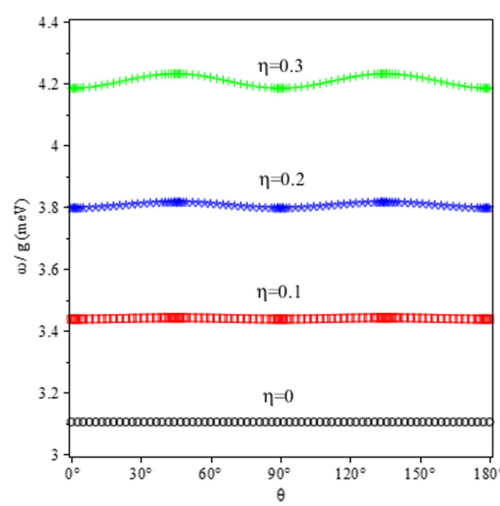


**Fig. 1** Angular dependence of the spin wave gap for different values of $\eta$. $H = 1.5\,T$

which can also be used to define the spin wave gap at zero magnetic field so that the gap is essentially determined by the magnetic anisotropy as we have expected. Pramanik et al. [15] have measured the first- and second-order uniaxial perpendicular anisotropy constants as a function of two different orientations of external magnetic field: in-plane (hard axis) direction and out-of-plane (easy axis) direction. The magnetic field $H$ is given in units of $g\mu_B = g \times 5.79 \times 10^{-2}\text{meV}/_T$. In the computation, in perpendicular magnetic field ($H \leq 1.5\,T$), the constants $H_1 \cong 2.809\,T$, $H_2 \cong 0.919\,T$, and $M_S = 6.2 \times 10^5 A/m$ were used for $L = 20$-nm-thick film at 2 K [15]. Although Pramanik et al. [15] have observed two- and fourfold anisotropy fields in their materials, however, they did not mention any measured in-plane term $H_{\text{in}}$, which breaks the fourfold symmetry. This attribution only adds to the magnitude of the spin wave gap; however, it is important for our calculations. The uniform perpendicular magnetized state becomes unstable by nucleation of magnetic domains with in-plane components of magnetization along different directions. The small in-plane anisotropies cannot be neglected for these materials. This occurs for two reasons. Firstly, because an easy plane magnet is unstable in two dimensions and hence, any small effect which produces stabilization has a dramatic effect and secondly, because the energy gap does not vary linearly with the in-plane anisotropy, but is magnified because it is determined by the geometric mean of the in- and out-of-plane anisotropy energies. Our theoretical results are well suited for their experimental data given [15], except for in-plane anisotropy. Figure 1 shows the spin wave gaps as a function of the angle $\theta$ between the applied field direction and the [001] axis for the dimensionless parameters $k = \frac{H_2}{H_1}$ and $\eta = \frac{H_{\text{in}}}{H_1}$. Experimentally, the applied field strength is $1.5T$ and $k = 0.33$ for various values of $\eta$. As can be seen, the variation in energy gap is due to the in-plane anisotropy presented in the film. One striking feature of this plot is that the amplitude of the energy gap variation increases gradually for the higher order $\eta$ in which the effect of exchange is not included. Figure 2 shows the plot of spin wave gaps as a function of the applied magnetic field with respect to the [001] axis for various values of $\eta$. Obviously, the spin wave gap calculated at zero temperature is almost constant below about $H \lesssim 0.056\,T$. At high fields ($H \gtrsim 0.056\,T$), the spin wave gap increases almost linearly with increasing magnetic field. This indicates that the film is uniformly magnetized. Besides, the spin wave gap increases with increasing in-plane anisotropy. As $\eta$ increases, the directional dependence of spin wave gap also increases periodically and the spin wave gap approaches that of (6). The anisotropy modes are significantly affected by changing the orientation of the magnetization with respect to the crystallographic axes. Due to the induced twofold

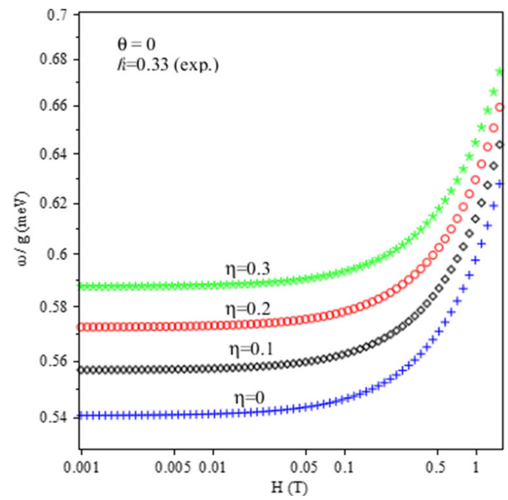


**Fig. 2** Field dependence of the spin wave gap for different values of $\eta$

symmetry a uniaxial in-plane anisotropy was found to be magnetoelastic [7].

We next concentrate to achieve a better understanding of the angular dependence of magnetization reversal in epitaxial $Cr_2Te_3$ thin films with in-plane anisotropy as well as perpendicular first- and second-order anisotropies. So far, we are only interested in the quantum physics. However, at this point, it is obvious how we make the transition from our quantum theory to the classical theory of spin waves. We choose the spherical coordinate system in which $\theta$ is the polar angle with respect to the film normal and $\varphi$ defines the in-plane orientations. We consider only external magnetic fields $H$ which are perpendicular to the plane of the film and thus parallel to the easy axis. The total energy of

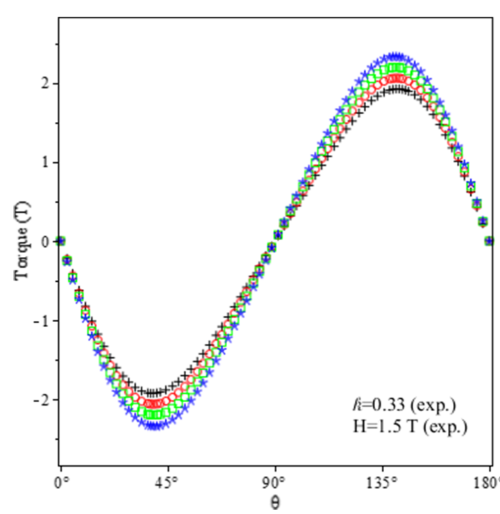


**Fig. 3** The torque moment as a function of $\theta$ for different values of $\eta$. (cross: $\eta = 0$; circle: $\eta = 0.1$; square: $\eta = 0.2$; asterisk: $\eta = 0.3$)

a uniformly magnetized film is written from the Hamiltonian (1). In (1), the $z$ component of the magnetization is $M$, and the magnitude of the magnetization is assumed to have a constant value of saturation magnetization, $M_S$. The total energy has minima at $\theta = 0$ and 180° for positive $K_1$, $K_2$, and $K_{\text{in}}$. Figure 3 shows the calculated torque moment as a function of angle $\theta$, at $H_1 = 2.809\,T$ and $k = 0.33$ for various $\eta$ values. On the one hand, a periodicity of 180° in the oscillation curve is clearly seen, which indicates that a fourfold magnetic anisotropy does exist as well as in-plane anisotropy.

The coherent switching behavior is calculated in a single-domain model by minimizing the total energy as a function of the in-plane angular orientation of the magnetization vector. It is not possible to closely fit the data using such a model since domain pinning effects are clearly occurring [15]. Nonetheless the calculations yield a reasonable estimate of $\eta$. A striking feature of the data is the strong twofold anisotropy which dominates over the fourfold anisotropy associated with the cubic symmetry of the film. Such a strong twofold anisotropy may originate in the very high strain induced by the film adopting the hcp/fcc structure [15]. Magnetic field dependence of the observed magnetization deviation of the $Cr_2Te_3$ film of 20-nm thick at 2 K is shown in Fig. 4 where the straight line is least-squares fit to the data to a constrained polynomial function, $y = -\left(H_1 + H_{\text{in}}\right)x - 2H_2x^3$, where $y = H$ and $x = M/M_S$. The fit gives $\eta \simeq 0.26$ in the range $-0.8\,M_S < M < 0.8\,M_S$. Obviously, we realized immediately that the fitting is fairly well, as shown by the solid line in the figure. Figure 5 shows the angular dependence of switching field as a function of angle $\theta$ between the direction of magnetic

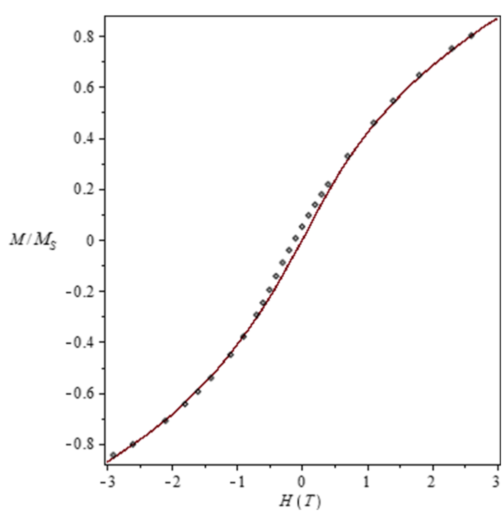


**Fig. 4** $M_S/M$ vs $H$. The squares are experimental data. The solid line is fitting curve according to the constrained polynomial function given in the text

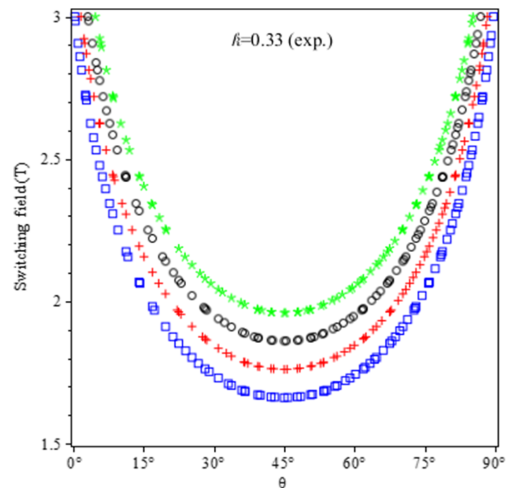


**Fig. 5** Angle dependence of the switching field for different values of $\eta$. $H = 1.5\,T$. (square: $\eta = 0$; cross: $\eta = 0.1$; circle: $\eta = 0.2$; asterisk: $\eta = 0.3$)

field and anisotropy axis. It is seen that the critical field is minimum at $\theta = 45°$. When the field is 45° from the easy axis, the reversal of magnetization takes place most easily. The critical field is then $\sim \frac{2K_i}{M_S}$ $(i = 1, 2, \text{in})$ for $\eta = 0.3$. The critical field becomes larger and larger at $\theta = 0$ and 90°. It has to be pointed out that the angle dependence of switching field shows a highly uniform reversal mode which is much closer to the well-known Stoner–Wohlfarth (SW) model [21]. We explain that this may be due to the low-domain propagation field in the thin film, which is much smaller than the nucleation field required to form the initial reversed nucleus. Therefore, the angle dependence of switching field is determined by the nucleation field of the initial, small reversed domain. However, Pramanik et al. [15] have observed that the switching fields are lower than the anisotropy field, and assumed one-dimensional defect model, in which they reported that the deviation from SW model may be related to the nucleation and domain wall pinning due to defects and inhomogeneities in their material.

In conclusion, our calculations confirm that the in-plane anisotropy contribution influence the spin wave gap of ultrathin films with small thickness at the absolute zero of temperature. The in-plane anisotropy should dominate in determining the spin wave gap by over an order of magnitude. It is important to rule out the possible effect of an applied field in determining the spin wave gap which validates the presented models. We also discussed for the angular variation of switching field by using a quasi-classical argument which is much closer to the Stoner–Wohlfarth model. It is obvious that our results show reversal modes to uniform rotation in which the nucleation of the reversed domain determines the switching field.

**Acknowledgements** The authors thank Tanmoy Pramanik, from The University of Texas, for the experimental data of $Cr_2Te_3$.

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