

# EXAMINATION OF THE EIGENVALUES LORENZ CHAOTIC SYSTEM

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## Abstract

In this paper, Classical Lorenz Equations are simulated using MATLAB/Simulink, by getting the graphical outputs performances of the equations are studied. The butterfly effect was seen in the expected output graphs. Classical Lorenz Equations were linearization and then Jacobian matrix was obtained by MATLAB software in embedded system and eigenvalues calculated. The numerous eigenvalues poles were seen on the right axis side of the complex plane. These poles are causes the unstable behaviors.

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**Keywords:** Lorenz Equation, Chaos, Stabilite, MATLAB, Simulink

## Introduction

Chaos is a well-known nonlinear phenomenon, and it is the seemingly random behavior of a deterministic system that is characterized by sensitive dependence on initial conditions. Besides, chaos is occasionally preferable, but usually intrinsically unpredictable as it can restrict the operating range of many physical devices and reduce performance (Hsiao, 2013).

Chaos theory was appeared as a scientific discipline at 1960's with Edward Lorenz, who has studied to model meteorological systems with Lorenz Equations in computer environment by using the data he collected to estimate the weather forecast (Uyaroğlu, 2009).

Chaos, a nonlinear phenomenon, has been intensively investigated in medical science, biology, and physics (Ma, 2011).

In these days, Chaos theory has successful applications in the fields such as secure communication, automatic control systems, laser physics and financial modeling (Uyaroğlu, 2009).

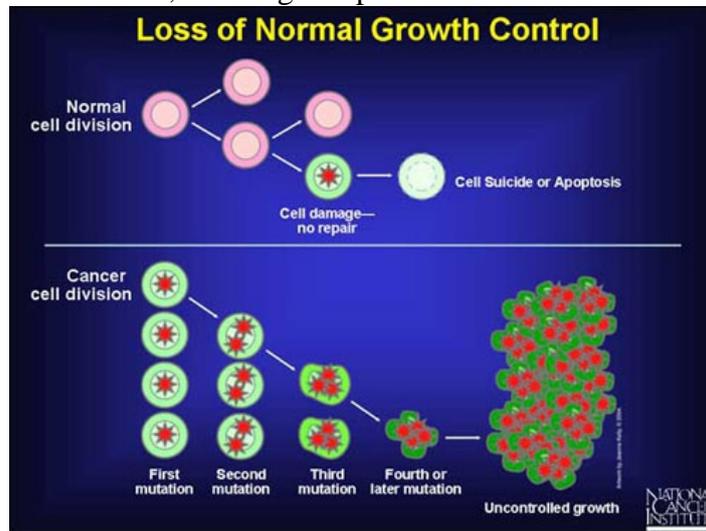
The problem of controlling chaos has recently attracted the attention of many investigators. Though chaos is a beneficial feature in some chemical or heat and mass transport processes (Ottino, 1989).

Many researchers have endeavored to find new ways to suppress and control chaos more efficiently. So far, many researchers have presented different types of controllers and control methods, e.g., linear state error feedback control (Uyaroğlu, 2013) (Wu, 2007), sinusoidal state error feedback control (Chai, 2007), variable substitution control (Wu, 2006), variable structure control (Yan, 2007), nonlinear feedback control (Chen, 2004), active control (Thang, 2009), and adaptive control (wei, 2007) have been successfully applied to chaotic systems (Uyaroğlu, 2013).

For the study of chaos theory and application of new or existing chaotic attractors reveal the chaotic systems and improve the dynamics of topological structure, besides the current system has an important place in terms of applicability (Uyaroğlu, 2009).

The chaos studies consist of the processes to improve the terminated chaotic behaviors or chaos behavior of systems to stabilize the structure, systems and methods for more efficient use of areas. For example, the behavior of cancer cells shows a chaotic structure in a shown Picture 1. The disease cancer occurs due to a group of cells exist differentially in the body, as

a result of excessive and uncontrolled proliferation. Normally, the growth and proliferation of cells is a normal regime. Accordingly, tissues and organs are able to do their duties as normal. However, these cells start to grow and replicate abnormally, this process give rise to the formation of masses called tumors (novartisonkoloji ). Here, the chaotic behavior should be terminated and the cell structure, showing cell proliferation must be stabilized.



**Picture 1:** Cancer Cell Division (Cancer)

In another example, a dishwasher water distribution nozzles engaging movement in the structure behaves as a dynamical chaos ensuring more efficient washing of the dishes has been provided (Chau, 2011).

In this study, taking advantage of the chaotic behavior of Lorenz equations with the help of MATLAB/Simulink simulation studies were obtained. The changes of variables was examined by obtaining the eigenvalues.

**Lorenz Systems**

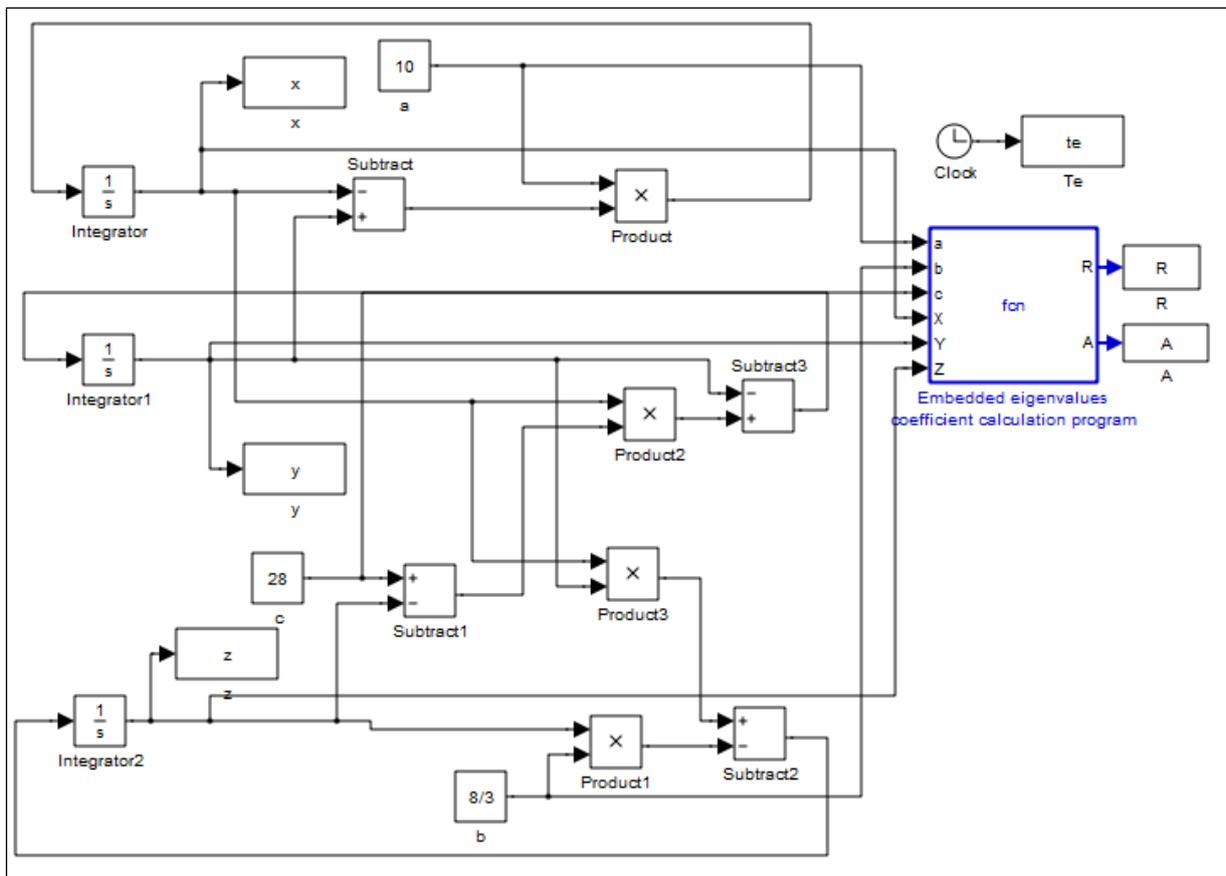
The Lorenz systems is described by the following nonlinear differential equations where x, y and z are state variables, and a, b and c are positive constant parameters when the typical parameter values for a Lorenz System are a=10, b=8/3 and c=28;

$$\begin{aligned}
 \dot{x} &= a \cdot (y - x) \\
 \dot{y} &= x \cdot (c - z) - y \\
 \dot{z} &= x \cdot y - b \cdot z
 \end{aligned}
 \tag{1}$$

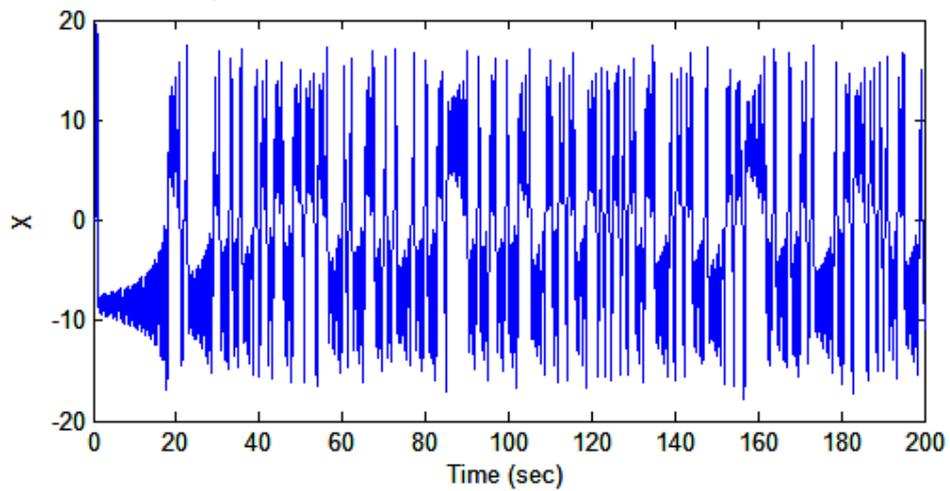
The Lorenz systems is described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ -28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}
 \tag{2}$$

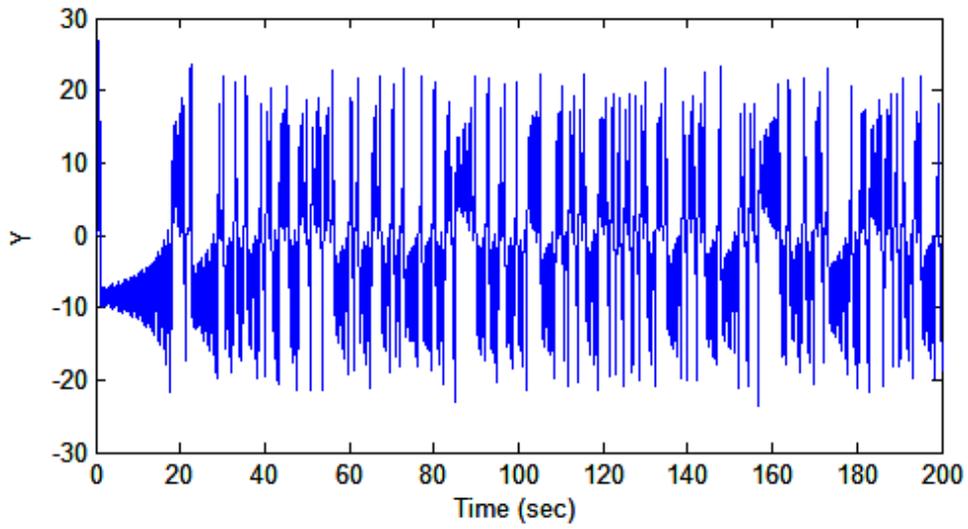
Using a MATLAB/Simulink model, as shown in Figure 1., the time series of the Lorenz chaotic system are shown in Figure 2, Figure 3 and Figure 4., the xy, xz, yz and xyz phase portraits are showing in Figure 5, Figure 6, Figure 7 and Figure 8 When  $x_0=0.001$ ,  $y_0=0.001$  and  $z_0=0$ .



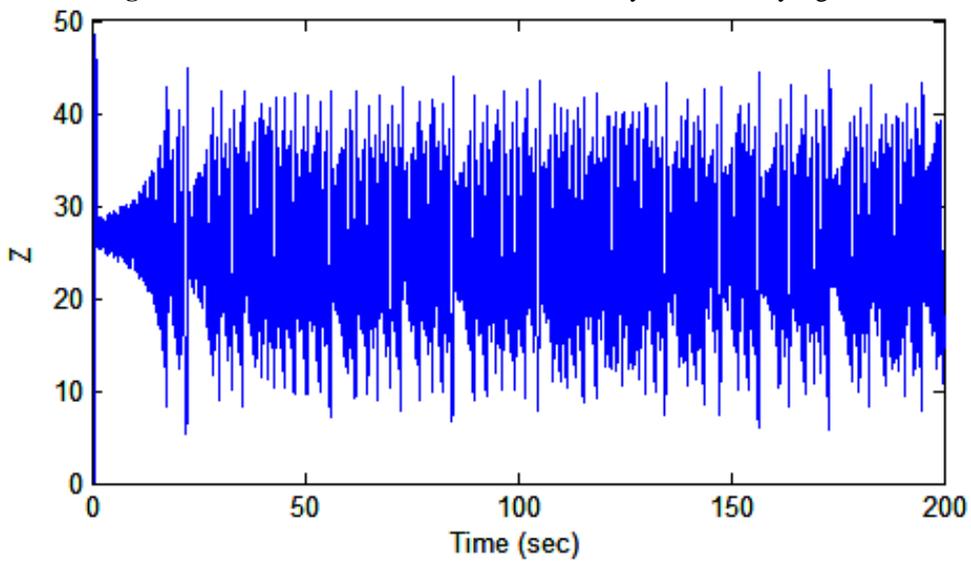
**Figure 1:** Lorenz MATLAB/SIMULINK chaos model



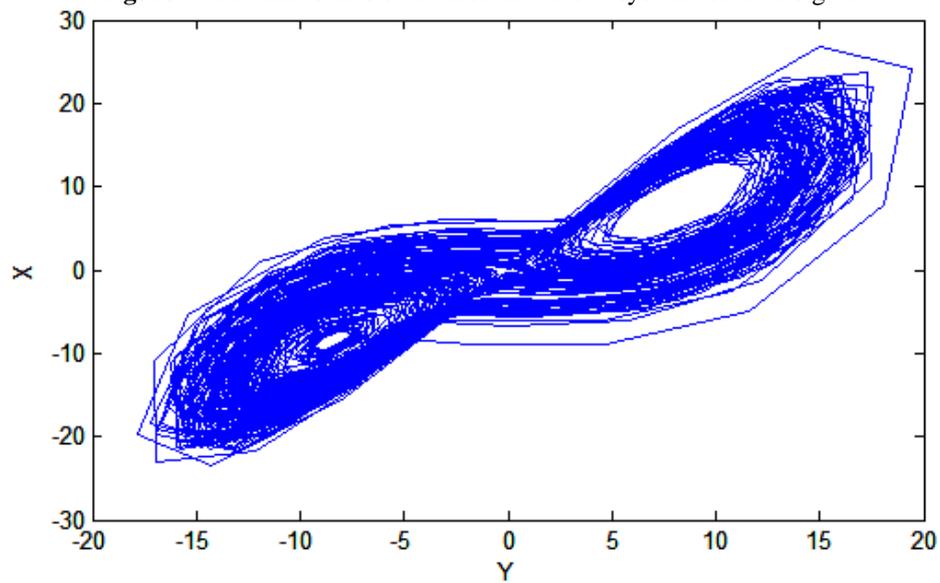
**Figure 2:** The time series of the Lorenz chaotic system for the x signal



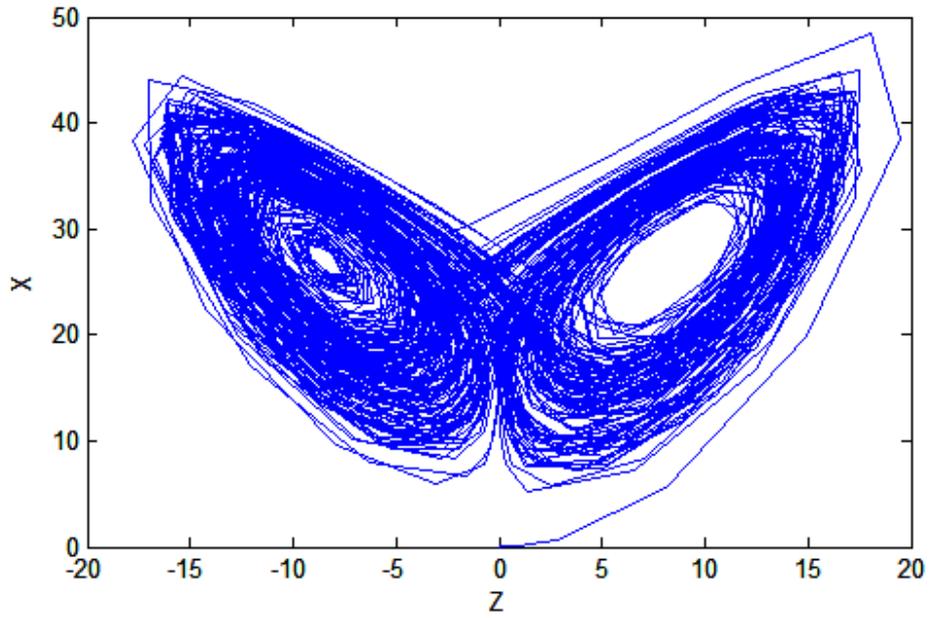
**Figure 3:** The time series of the Lorenz chaotic system for the y signal



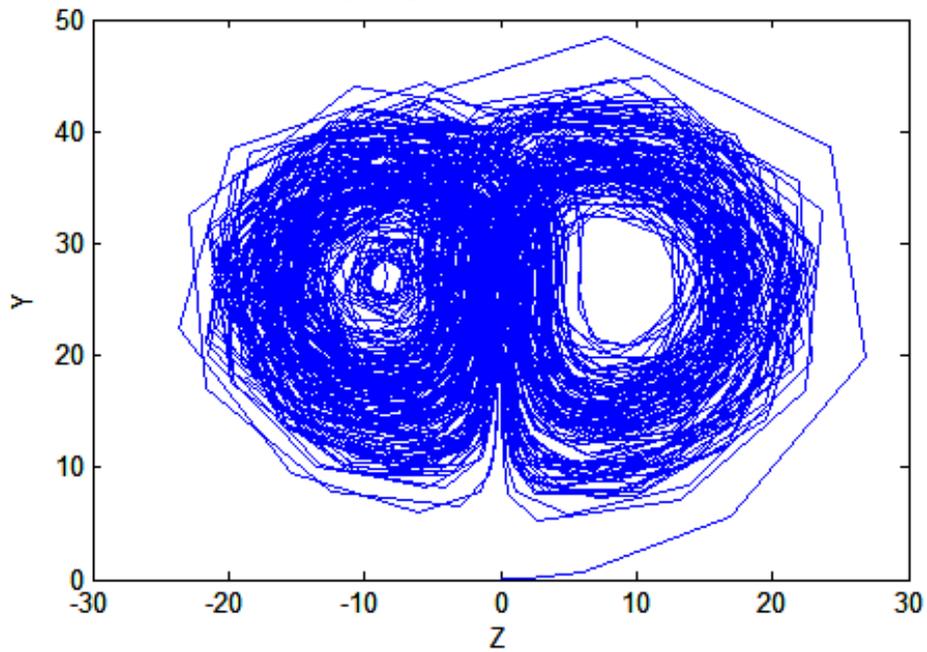
**Figure 4:** The time series of the Lorenz chaotic system for the z signal



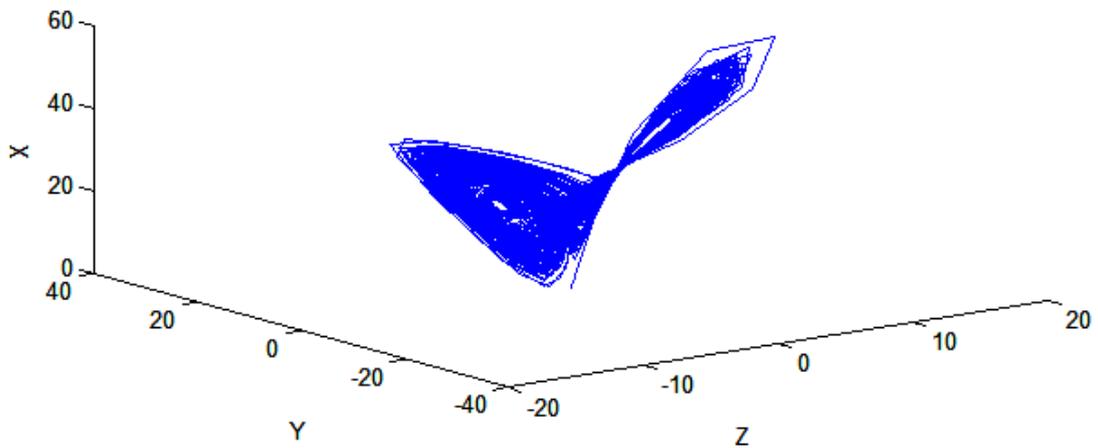
**Figure 5:** xy phase portait of the Lorenz Systems



**Figure 6:** xz phase portait of the Lorenz Systems



**Figure 7:** yz phase portait of the Lorenz Systems



**Figure 8:** xyz phase portait of the Lorenz Systems

Phase portraits are showing in Figure 5, Figure 6, Figure 7 and Figure 8. The butterfly effect was seen in the expected output graphs.

The stable and unstable points of the sytem variables was examined by obtaining the eigenvalues. The system is linearization to accomplish eigenvalues and then the Jacobian Matrix is obtaing by using the first terms of linear elements.

$$\begin{aligned} f_1 &= \dot{x} = a \cdot (y - x) \\ f_2 &= \dot{y} = x \cdot (c - z) - y \\ f_3 &= \dot{z} = x \cdot y - b \cdot z \end{aligned} \tag{3}$$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ c - z & -1 & -x \\ y & x & -b \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 \\ 28 - z & -1 & -x \\ y & x & -8/3 \end{pmatrix} \tag{4}$$

The eigenvalues are found by solving the characteristic equation,

$$A = |\lambda I - J| = \begin{vmatrix} \lambda + 10 & -10 & 0 \\ -28 + z & \lambda + 1 & x \\ -y & -x & \lambda + (8/3) \end{vmatrix} = 0 \tag{5}$$

Using eig (A) MATLAB code for eigenvalues of Lorenz chaos system.

Initial eigenvalues of  $\lambda_1 = 22.8277$ ,  $\lambda_2 = -11.8277$ ,  $\lambda_3 = 2.6667$  for  $x_0=0.001$ ,  $y_0=0.001$  and  $z_0=0$ . Initial eigenvalues are shown in Figure 9 (a, b, c,). All of eigenvalues are shown in Figure 10.

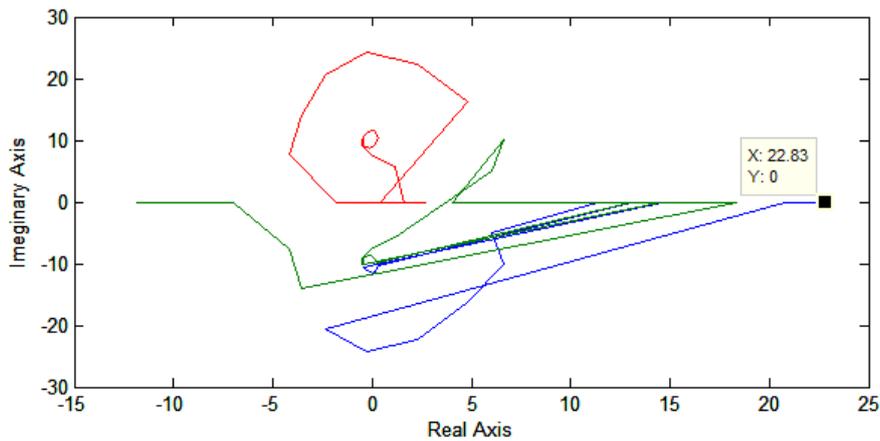


Figure 9(a): Eigenvalues coefficients in complex plane

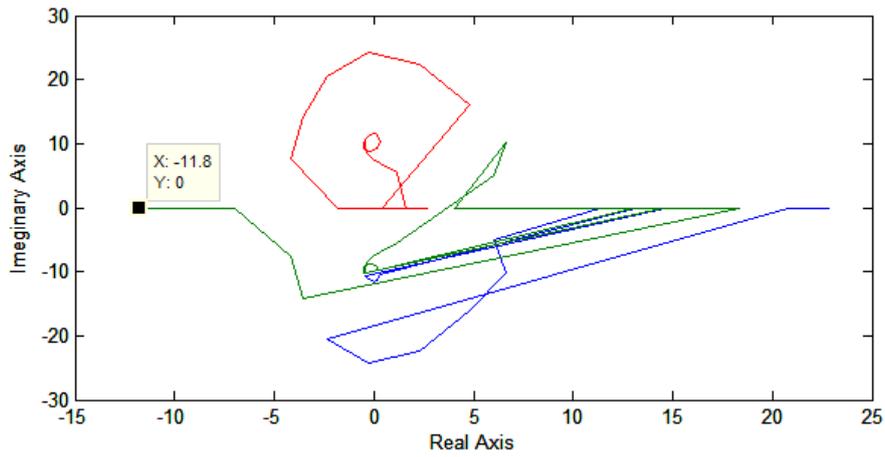


Figure 9(b): Eigenvalues coefficients in complex plane

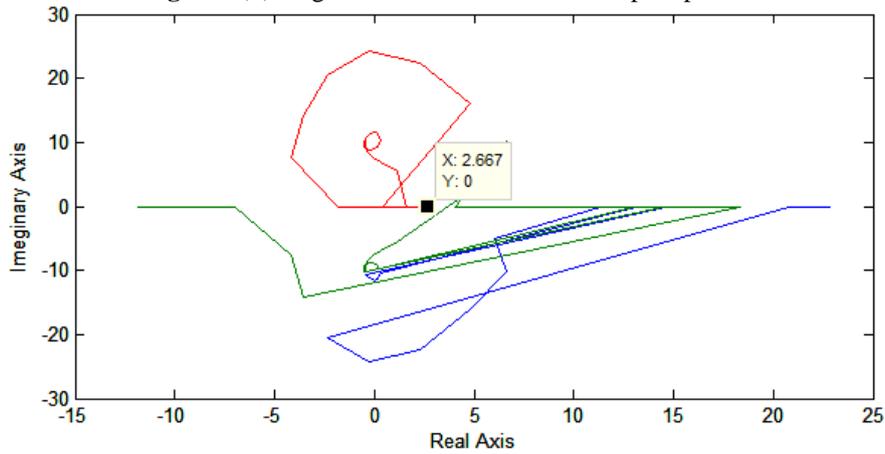


Figure 9(c): Eigenvalues coefficients in complex plane

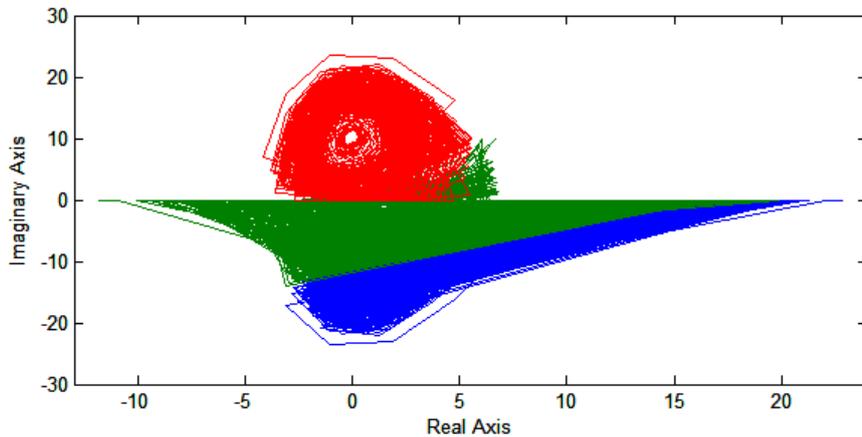


Figure 10: Eigenvalues coefficients in complex plane

### Conclusion

The aim of this paper is to investigate the stable of the Lorenz chaotic systems. It is seen in Figure 10 that the eigenvalues coefficients in complex plane are located on the right side of the complex plane axis. As a result of unstable of the system is causes the nonlinear behaviors. The eigenvalues of the system is show in Figure 9(a,b,c). The eigenvalues given referance 5 were achived same values that in Figure 9(a,b,c). The various of eigenvalues coefficients in compex plane are revealed a chaotic behavior. In addition the butterfly effect was seen in the output graphs.

## References:

- Hsiao, F. H. (2013) Robust  $H_{\infty}$  Fuzzy Control of Dithered Chaotic Ssystems. *Neurocomputing* Vol: 99, pp.509-520.
- Uyaroğlu, Y. Gündüz, S., Yiğın, İ. H. and Keskin, H. (2009) Simulation and Characterizaion of Lorenz Equations in Matlab and Simulink Enviroment at Chaos Theory. 5. *Uluslar arası İleri Teknolojiler Sempozyumu (IATS'09)*, 13-15 Mayıs, Karabük, Türkiye.
- Ma, C. Y., Liu, J. H. and Wang, C. L. (2011) Chaos of a Power System Model and its Control. *Journal of Vibration and Control*, Vol: 18(14), pp. 2176–2185.
- Ottino, J. M. (1989) Kinematics of Mixing: Stretching, Chaos and Transport, *Cambridge University Press*, Cambridge.
- Uyaroğlu, Y. and Cevher, B. (2013) Chaos control of Single Time-Scale Brushless DC Motor With Sliding Mode Control Method. *Turk J Elec Eng & Comp Sci.*, Vol: 21, pp.649 – 655.
- Wu, X., Chen, G. and Cai, J.(2007) Chaos Synchronization of the Master-Slave Generalized Lorenz Systems Via Linear State Error Feedback Control. *Physica D: Nonlinear Phenomena*, Vol. 229, pp. 52–80.
- Cai, J.P., Wu, X.F. and Chen, S.H. (2007) Synchronization Criteria for Non-Autonomous Chaotic Systems via Sinusoidal State Error Feedback Control. *Physica Scripta*, Vol. 75, pp. 397–387.
- Wu, X.F. and Wang, M.H. (2006) Robust Synchronization of Chaotic Lur'e Systems Via Replacing Variables Control. *Interna-tional Journal of Bifurcation and Chaos*, Vol. 16, pp. 3421–3433.
- Yan, J.J., Hung, M.L., Lin, J.S. and Liao, T.L. (2007) Controlling Chaos of a Chaotic Nonlinear Gyro Using Variable Structure Control. *Mechanical Systems and Signal Processing*, Vol. 21, pp. 2515–2522.
- Chen, M., Zhou, D. and Shang, Y.(2004) Nonlinear Feedback Control of Lorenz System. *Chaos, Solutions and Fractals*, Vol. 21, pp. 295–304.
- Tang, R.A., Liu, Y.L. and Xue, J.K. (2009) An Extended Active Control for Chaos Synchronization. *Physics Letters A*, Vol. 373, pp. 1449–1454.
- Wei, D.Q., Luo, X.S., Wang, B.H. and Fang, J.Q. (2007) Robust adaptive dynamic surface control of chaos in permanent magnet synchronous motor. *Physics Letters A*, Vol. 363, pp. 71–77.
- [http://www.novartisonkoloji.com.tr/Default.aspx?sayfa=hasta\\_kanser\\_nedir](http://www.novartisonkoloji.com.tr/Default.aspx?sayfa=hasta_kanser_nedir).
- <http://www.cancer.gov/cancertopics/cancerlibrary/what-is-cancer>.
- Chau, K.T. and Wang, Zheng (2011) Chaos in Electric Drive Systems Analysis. *Control and Application. John Wiley&Songs (Asia) Pte. Ltd.*