An investigation of understanding of preservice elementary mathematics teachers (PEMT) about data displays

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Understanding to what extent preservice teachers are capable of conceptual and procedural knowledge of data displays is the aim of this study through analyzing a part of data collected during an independent study course. Change in the middle school curriculum in Turkey necessitates the study of examination of understanding of PEMT about statistics. Therefore, this study is significant in supporting the needs of teacher educators as well as it contributes to the consequences of curriculum efforts. In order to possess an understanding of statistical concepts for preservice mathematics teachers, they must have both conceptual and procedural knowledge (Hiebert & Lefevre, 1986), which is the main concern of this study.

Keywords: PEMT, data displays, conceptual knowledge, procedural knowledge.

INTRODUCTION

In our revised curriculum, which started to be instructed in middle-level schools in Turkey in September 2013, statistics has been the most emphasized subject. It was considered as a separate learning area named as data handling and it was included in all grades from the 5th grade through the 8th grade. However, the content of probability was reduced compared to previous curriculum, and it is placed into the 8th grade level only with a basic understanding of probability. These changes in the new curriculum could be identified as reflection of Moore's (1997, as cited in Biehler, Ben-Zvi, Bakker, & Makar, 2012) recommendation emphasizing that curriculum needs more statistics and less probability while leaving the deeper conceptual knowledge to the high-school level. The earlier studies showed that PEMT have less comprehension of statistics and probability compared with the other learning areas of curriculum, that I,; they found probability and statistics subjects difficult to teach especially because of their lack of content knowledge in probability and statistics (Quinn, 1997; Stohl, 2005). Contemporary efforts are addressing the same issue as well, so that teacher education should be enhanced through giving an attention to statistics and probability teaching for mathematics teachers (Stohl, 2005; Jones & Thornton, 2005).

Change in the elementary school curriculum necessitates the study of examination of understanding of PEMT about probability and statistics. It should be understood whether preservice elementary teachers have both conceptual and procedural levels of understanding of probability and statistics in order to teach them (Star, 2005). Thus, this study is significant in the above needs of the Turkish mathematics education literature as well as it contributes to the consequences of curriculum efforts and will be a light for future considerations of this issue. This study is part of a relatively larger study which was conducted by the researcher in spring semester in 2014. The research questions examined here are as follows: (a) to what extent are PEMT capable of conceptual and procedural knowledge of data displays? (b) what are the main strengths and weaknesses of PEMT in data displays?

REVIEW OF RELATED LITERATURE

Statistical knowledge for teaching can be interpreted under the framework of mathematical knowledge for teaching. This framework has two main dimensions for mathematical knowledge for teaching: first, subject matter knowledge which includes common content knowledge (CCK), specialized content knowledge (SCK) and knowledge at the mathematical horizon; second, pedagogical content knowledge which includes knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of curriculum (Hill, Ball, & Schilling, 2008). From the statistics point of view, CCK is considered as computing and interpreting the most frequent measures of central tendency; SCK is considered as special for teaching as which is best for which statistics term; horizon knowledge is considered as working on populations will eventually emerge the working on samples, for example. For the second dimension, KCS can help teachers to catch the common strategies which students use in developing students' statistical reasoning; KCT deals with the content-specific strategies like knowing how to explain arithmetic mean as a fair share or as a balance point; and knowledge of curriculum can help teachers about structural properties that a curriculum possess (Groth, 2012). Therefore, Groth (2012) has developed a framework for combining above terminology and suggested the figure in his paper.

Based on the efforts in conducting the course which Groth (2012) was teaching, namely as Statistical Knowledge for Teaching (SKT), he has developed the framework for SKT, while adding two new constructs to the statistical knowledge for teaching framework, one of which is key developmental understandings and the second one is pedagogically powerful ideas. Key developmental understandings were defined as "cognitive landmarks in the learning of fundamental ideas needed to understand content" (Simon, 2006, as cited in Groth, 2013). Pedagogically powerful ideas can be defined as ideas that occur as the result of transforming key developmental understandings into ideas that facilitate students' learning of the key developmental understandings. Groth (2013) in his hypothesized framework relates these two dimensions with the other existing dimensions of mathematical knowledge of teaching.

According to above framework, subject matter knowledge needed for statistics and probability teaching is the concern of this study and its sub-dimensions such as conceptual and procedural knowledge types should be integrated to it as well. The terms were introduced by Scheffler (1965), but expanded by Hiebert and Lefevre (1986) and Star (2005), where "conceptual knowledge is characterized most clearly as knowledge that is rich in relationships, like a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (p. 3). They also categorize the conceptual knowledge as primary and reflective. The primary level of conceptual knowledge is formed as in "the relationship connecting the information is constructed at the same level of abstractness (or at a less abstract level) than that at which the information itself is represented" (Hiebert & Lefevre, 1986, p. 5). The reflective level of conceptual knowledge is constructed in 'a relationship which requires a higher, more abstract level than the pieces of information they connect' (p. 5). Apart from conceptual knowledge, Hiebert & Lefevre (1986) also explains the procedural knowledge in two types: "one kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols; the second kind of procedural knowledge consists of rules or procedures for solving mathematical problems" (p. 7). In order to develop an understanding of statistical concepts for PEMT, they must have both conceptual and procedural knowledge (Hiebert & Lefevre, 1986). Ball (1988) described the subject matter knowledge similar to above; she names the procedural knowledge as substantive knowledge, which refers to "understanding of particular topics, procedures, concepts and relations among them" (p. 4), and secondarily, knowledge about mathematics is named in place of conceptual knowledge. Hence, this type of categorization fits with the above expressions and summarizes our theoretical framework.

METHODOLOGY

This study has a phenomenological approach which is categorized by one of the qualitative research designs (Creswell, 2007) since researcher tries to understand the shared experiences and understandings of a group of participants on data displays and their weaknesses/strengths as a phenomena.

In order to investigate its research questions, data were collected through face-to-face interviews (later transcribed verbatim). 23 participants from PEMT were interviewed voluntarily, 12 of whom are 4th year students and the rest are in their 3rd year of elementary mathematics teacher education. They have already attended to a statistics and probability course (which is a must course in their undergraduate education) in their second year and it includes the probability and statistics subjects in a very advanced and a theoretical form. Apart from that, six participants had taken the "methods for statistics teaching" (which is a fourth year must course in one of the universities studied) before the instrument and they specified especially that they learned statistics through that course.

In order to investigate the understanding of PEMT's knowledge of statistics and probability subjects framed by elementary school curriculum, an instrument was used. It involves open-ended questions as well as multiple-choice items which measure both procedural and conceptual types of knowledge required for understanding of statistics and probability. Specifically, it involves statistics questions related to measures of central tendency (mean, median and mode), measures of variation (range) and data displays; and probability questions related to probability of a basic event, certain, impossible and equally-likely events, theoretical and experimental probability. Some items are presented at the end of the paper as Appendix. The instrument is organized as most of the questions were taken from two tests written by Diagnostic Teacher Assessments in Mathematics and Science developed at the University of Louisville (CRiMSTeD- Center for research in Mathematics and Science Teacher Development) with their permission to use. There is also one question taken from Jacobbe's (2007) dissertation. These assessments have established high levels of reliability and validity (Bush et al, under review, as cited in Jacobbe, 2007). Since the instrument was implemented to the participants as face-to-face interviews, it is necessary to mention about the interviews summarized below.

Each interview is composed of three sections. In the first section, participants were asked questions related with their choice of being elementary mathematics teacher, interest of mathematics and mathematics teaching, subjects which they know as best and least covered in middle school mathematics curriculum, comments on change on curriculum regarding the statistics and probability subjects, interest of learning/using technological tools/materials needed for teaching and comments on test items at the end of the interview. In the second section, participants were directed to questions related with the measures of central tendency, measures of variation and probability, such as 'how do you define mean?' or 'what does mode of a group of data tell you?' in procedural and conceptual knowledge levels. Thus, the findings of this part were the results of test items according to the subjects. In the last part, participants were given the test and they were expected to solve open-ended questions as orally. Implementing the instrument as in this way provides us in order to learn how preservice mathematics teachers understood probability and statistics, more specifically their conceptual and procedural way of understanding on these subjects.

The last two sections consider the content knowledge of participants while taking its different types as procedural and conceptual into consideration comparatively. That is, these sections compensate for each other in order to investigate the understanding of participants on the subjects which analyzed.

The analysis of the data gathered from first and second part of the interviews was done according to themes and codes specified before the implementation. The evaluation of the data gathered from the instrument was performed according to a rubric which was prepared for only open-ended items. The evaluation of open-ended items was done in only three categories: full response, incomplete response and wrong response. A full response addresses that participant gives the best explanation using the right terminology and expected logical foundations in order to rationalize the subject whereas an incomplete response addresses that participant does not give a fully satisfying response or s/he cannot rationalize his/her response. A wrong response means that participant gives a completely wrong response without making any logical explanation or rationalization by using his/her understanding of the subject.

FINDINGS

Although the methodology part mentions about the parent study of the study framed here, only the findings for data displays will be presented in order to answer the research questions outlined above. The instrument included 22 items and 11 items were associated with data displays. The responses given to each item and the subject related with the item are given below in the table:

Most of the participants did not know about most of the graphical representations which directed as questions through the instrument. For example, all of the participants do not seem to have any idea about box-and-whiskers plot and they couldn't catch the me-

Question	Type	Related Subject	Ratio of Achievement
Item 1	Multiple Choice	Stem-and-leaf display	2 correct, 1 wrong, 20 of 23 have no information about topic.
Item 2	Multiple Choice	Graphical representations of data	2 correct, 5 have no information about the subject, 16 of 23 are wrong.
Item 3	Multiple Choice	Scatter-plot	13 of 23 are correct.
Item 4	Open-ended	Box-and-whiskers plot	All of them have no information about topic, and they analyzed graphic with their own understanding, and sometimes come up with correct answer. Hence, they answered incomplete.
Item 5	Open-ended	Biased graphic displays	2 of 23 responded full, 1 of 23 responded false, and 20 of them responded incomplete.
Item 6	Open-ended	Circle graph	13 of 23 responded full, 2 of them responded wrong, 8 of 23 responded incomplete.
Item 7	Multiple Choice	Line graph	10 of 23 are correct.
Item 8	Multiple Choice	Line graph	22 of 23 are correct.
Item 9	Multiple Choice	Frequency table	21 of 23 are correct.
Item 10	Open-ended	Line graph, bar graph, categorical variable	10 of 23 responded wrong, 10 of 23 responded full, 3 of 23 responded incomplete.
Item 11	Open-ended	Frequency distribution, mean, median, mode, range, normal distribution, data displays	8 of 23 responded incomplete, 15 of 23 responded full.

Table 1: Findings per item with respect to the subject and type

dian and percentiles from the representation given in the item 4 (in Appendix). Most of the participants do not know of stem-and-leaf display, either, as realized from the findings of item 1 (in Appendix) in the test. The item 2 is another item which has only 2 correct responses. Item 2 was asking the best description of the distribution of achievement test scores for all 4th graders in a school while choosing among scatter plot, box-and-whiskers plot, line graph and circle graph. This finding is expected since box-and-whiskers plot is not known by the participants so as in item 4.

The scatter-plot question (item 3) has a half success by the participants. Most of the participants cannot relate each dot with two axes onto the graph, many of the correct responses were given with an unclear explanation. Although, all of the participants could realize the difference between two different graphs with the same data, they couldn't explain its consequences correctly and their responses show a misunderstanding or inadequate knowledge of biased graphical data displays. Item 6 asks for reasons of mistakes made by a student who draws a circle graph of a given set of data and how to overcome those mistake. Most of the participants could easily found the mistake, however, nearly half of them could present ways to overcome it.

Item 7 (in Appendix) is having one of the most wrong respondents which is also an unexpected result compared with the result of other line graph question, item 8. Item 8 asks the true alternative for the difference of average salaries taken by university or high school graduated workers and it includes two line graphs on the same display. Item 9 presents the frequencies of data as a rotated bar graph and asks the true alternative among the other 3 false alternatives like in the previous item. The responses for this item are mostly correct and we can say that the most achieved items which the participants responded with a higher rate than others were the 8th and 9th items. Another item which nearly half of the participants responded correctly is the item 10 (in Appendix) since they could realize the importance of having a categorical variable (like in this item) for a bar graph in this item. The item 11, which was taken from the Jacobbe's (2007) dissertation, is the longest question in the instrument. Nearly two-thirds of the participants responded as full, the rest of them could neither grasp the dot plot display nor explain their choice with a satisfying rationale.

DISCUSSION

The findings of this study show some aspects mentioned in the theoretical framework for preservice mathematics teachers' understandings described by Groth (2013), Hiebert & Lefevre (1986) and Ball (1988). It can be said that the questions directed to participants during the interview, they could not show their conceptual knowledge about data displays since their answers were mostly in procedural knowledge base. In general, PEMT has a high achievement in procedural level of knowledge for data displays.

It is also worth to mention that the responses given to the items in the instrument shows also resemblance to the findings through interviews which investigates the understanding of the statistical concepts, specifically measures of central tendency and measures of variation. For definitions of mean, median and mode, participants seem to not know the difference between calculations and meanings of them, as emphasized before (McGatha, Cobb, & McClain, 1998, as cited in Jacobbe, 2007). This can be discussed that they don't know the foundations under mean, median and mode and can be explained as there is a difference between knowing how something is calculated and knowing why it is (Hiebert & Lefevre, 1986).

Moreover, based on the findings of the interviews, it can be claimed that participants has a high achievement in procedural level of knowledge for measures of central tendency. They mostly know concepts; but, most of the participants have difficulty in answering questions necessitating conceptual knowledge, which are connected with the subjects of meanings of measures of central tendency as related with data displays, difference or relation between those. It can be argued that participants have not enough ability to connect what they know about measures of central tendency and the associated data displays; besides, they have not a higher-order comprehension needed for knowledge answering to the questions (Ball, 1988; Hiebert & Lefevre, 1986; Groth, 2013).

When we consider the possible reasons of why conceptual knowledge of PEMT have been less-developed compared with procedural knowledge, the courses offered for teacher candidates during their university education are like 'recipe-type' or 'rule-bound' courses which only deals with the calculations and lead preservice teachers to memorize the subjects while underestimating the logic behind it, as Shaughnessy (1992) stressed out previously. He claims also that preservice teachers lack of opportunity to develop their stochastic reasoning in university courses due to their misunderstandings about statistics. Nearly half of the participants have stressed during interviews that they feel themselves not knowing very well about statistics although they have taken a course namely as "statistics and probability" which they took in their 2nd year. The other half of the students have mentioned that they have a course about "teaching probability and statistics in elementary level". However, unless they learned about statistics very well, they do not feel to be able to teach it. Hence, they first need to know it, as they expressed and eventually corresponds with the arguments of Shaughnessy (1992). Although it was not one of the interview questions, most of the participants mentioned about their complaints about "statistics and probability" course. They emphasized that the course have not included any practical implementation to a real-life example, but was mostly theory-based and proof-based. Therefore, most of them specified that they could not learn much about statistics or probability subjects in this course, for example, they did not meet even with a boxplot display during the course.

In brief, this study discussed the understanding of PEMT for statistics, specifically the issue of data displays. Findings implied that content knowledge assessed by the items in the instrument have two dimensions, procedural and conceptual knowledge, as discussed clearly by the researcher previously in the review of related literature part above (Hiebert & Lefevre, 1986; Ball, 1988; Groth; 2013) and corresponds to the framework which was bounded.

The implications of this study will be enlightening to the future research for the understanding of PEMT in Turkey. The discussion of the findings can have an impact on teacher education programs in the universities. While some universities have currently specific courses related with pedagogical content knowledge for statistics and probability including content knowledge needed for those subjects as well, some of them have not. As specified before, PEMT are not ready for teaching statistics because of lack of statistical knowledge (Greer & Ritson, 1994). Therefore, as the participants of this study stressed out that they should learn statistics very well while learning methods of teaching statistics. Having such consequences, this study can have positive influences on the development of elementary mathematics education programs, and might affect the perspectives of teacher educators, who are responsible for educating the teachers, as well.

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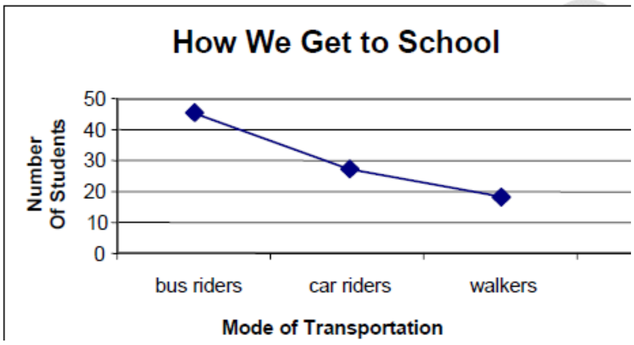
APPENDIX: SAMPLE ITEMS FROM THE INSTRUMENT

Item 7: Which of the following best describes a line graph?
a. A graph that visually represents the median, the quartiles, and the smallest and largest values of a data set.
b. A graph consisting of a horizontal number line with data points represented by X's.
c. A graph consisting of points, one for each item being measured. The two coordinates of a point represent the measures of two attributes of each item.
d. A graph with a vertical and horizontal axis that is primarily used to show changes over time.

Item 10: A student collected data from 90 fourth grade students about how each student traveled to and from school. The student created the following graph. How would you respond to this student?



Item 1: The students in a sixth-grade class were timed to the nearest second for a run around the school's gymnasium. The times for the class are listed below in a stem-and-leaf plot. Which of the following is true?

2	4 8
3	1 3 4 5 6 7 8
4	1 2 3 7 7 8
6	0 1 4 8 9

a. The lowest time was 28 seconds b. Half of the students had times above 41 seconds
c. The highest times was 60 seconds d. 50% of the students had times below 38 seconds

Item 4: The box-and-whiskers plot below represents the test scores of three classes on the same test.
a. Which class performed the best and which class performed the worst.
b. Provide justifications for your choices with data from the box-and-whiskers plots.