MOTION OF KLEIN-GORDON PARTICLE IN ANISOTROPIC BIANCHI-I TYPE UNIVERSE

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Abstract: *Relativistic scalar particles are described by Klein-Gordon equation. In the present study we solve KG equation in Bianchi-I type universe via Teleparallel theory. We obtained solutions in terms of Whittaker function and oscillator behavior of the scalar particles.*
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1. Introduction

An alternative way of studying the gravitational interactions is Teleparallel theory (TPT). In this theory, in contrast to general relativity (GR), torsion disappears instead of curvature. Also, the TPT uses the Weitzenböck geometry, whereas GR bases on Riemann spacetime. Gauge transformation and gauge invariance notions are known as the gauge theories and the gauge theory dates to pioneering work of Weyl in 1918 who has tried to unify the electromagnetism and gravitational interactions [1-2]. The TPT also introduced as a gauge theory in the beginning. Later, it has been developed by Einstein as a new method known as Absolute Parallelism (AP) geometry [3]. Then, Möller analyzed Einstein's approach and investigated gauge theories for gravitation and Lagrange formulation of TPT [4]. In 1979, Hayashi and Shirafuji also studied on TPT and they interpreted the TPT as a gauge theory for the translation group [5]. Nevertheless, these two theories conclude with the similar results and have achieved an increasing interest compared to different formulations of gravitational interactions. Therefore, this is only a matter of agreement that the existence of gravitational fields creates a curvature or torsion in the geometry of the spacetime.

In this work, we study the motion of the scalar bosons in an anisotropic Bianchi-I type universe by using TPT formalism. In Sec.2 we briefly give the Notion and definitions of TPT. In Sec.3. we obtain exact solutions and discuss the harmonic oscillation behavior of the scalar particles. Finally, in Section 4 we briefly discuss the results. Throughout the paper Latin and Greek indexes will be used to represent the Minkowski space and Weitzenböck spacetime, respectively.

The anisotropic Bianchi-I type spacetime model is represented by the following metric [6]

$$ds^2 = C_0^2(t)(-dt^2 + dz^2) + dx^2 + dy^2 \tag{1}$$

where the metric scale is $C_0(t) = bt$.

2. Teleparallel Theory: Notations and Definitions

As we stated in the above the TPT description of gravity become the gauge theory of translation group [2]. The TPT theory uses non-trivial tetrad fields and the Weitzenböck connections are derived from tetrads as follows [2]

$$\dot{\Gamma}^{\rho}_{\nu\mu} = h^{\rho}{}_{a}\partial_{\mu}h^{a}{}_{\nu} \tag{2}$$

For the tetrad $h^{a}{}_{\mu}$, the spacetime is related to tangent space metrics as

$$g_{\mu\nu} = h^{a}{}_{\mu}h^{b}{}_{\nu}\eta_{ab} \tag{3}$$

where η_{ab} is the metric tensor of Minkowski spacetime. The torsion of the Weitzenböck connections is defines ad below

$$\dot{T}^{\rho}{}_{\mu\nu} = \dot{\Gamma}^{\rho}_{\nu\mu} - \dot{\Gamma}^{\rho}_{\mu\nu} \tag{4}$$

and contortion of the Weitzenböck torsion is given by

$$\dot{K}^{\rho}{}_{\mu\nu} = \frac{1}{2}\left(\dot{T}_{\mu}{}^{\rho}{}_{\nu} + \dot{T}_{\nu}{}^{\rho}{}_{\mu} - \dot{T}^{\rho}{}_{\mu\nu}\right) \tag{5}$$

3. Solution of the Klein-Gordon Equation in Teleparallel Gravity

The GR formulation of the Klein-Gordon equation that is representing the relativistic spinless particles is given by the following equation [6]

$$g^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}\Phi - (m^2 + \xi R)\Phi = 0 \tag{6}$$

where, ∇_{α} is covariant derivative, R is the scalar curvature and ξ is the dimensionless coupling constant which will be set to zero for our case. The scalar field is described with the Klein-Gordon equation and it should couple to torsion, namely the Klein-Gordon equation in the TPT is given as [7]

$$\{\partial_{\mu}\partial^{\mu} + (\dot{\Gamma}^{\mu}{}_{\lambda\mu} - \dot{K}^{\mu}{}_{\lambda\mu})\partial^{\lambda} + m^2\}\phi = 0 \tag{7}$$

By using the metric (1), the non-vanishing Weitzenböck connections and contortions are found to be

$$\dot{\Gamma}^{0}{}_{00} = \dot{\Gamma}^{3}{}_{30} = \frac{1}{t} \tag{8}$$

$$\dot{K}^{0}{}_{33} = \dot{K}^{3}{}_{03} = -\frac{1}{t} \tag{9}$$

where the dot represents the Weitzenböck spacetime quantities. With the insertion of these expressions into the Eq. (7), we arrive

$$\left[\frac{d^2}{dt^2} + \left(k_x^2 + k_y^2 - m^2\right)b^2t^2 - k_z^2\right]\varphi = 0 \tag{10}$$

where we defined $\phi = \varphi(t)\, e^{i\vec{k}\cdot\vec{x}}$.

Definition: The general solutions of the differential equation given in the following form

$$x^2y'' - [ax + b]xy' + [\alpha x^2 + \beta x + \gamma]y = 0 \tag{11}$$

are given by [8]

$$y = x^{-\frac{1}{2}b}e^{-\frac{1}{2}ax}M_{\frac{2\beta-ab}{2\rho},\mu}(\rho x) \tag{12}$$

Then, solution of equation (10) is found in terms of the Whittaker functions as follow

$$\varphi(t) = M_{0,\sqrt{\frac{1}{4}+k_z^2}}\left(2b\sqrt{m^2 - k_x^2 - k_y^2 t}\right) \tag{13}$$

4. Oscillatory Behavior of the Solutions

In order to find the frequency spectrum of the scalar particles we can simulate the obtained equation (10) to the equation of the harmonic oscillator as follow

$$\left[\frac{d^2}{dt^2}+\omega^2(t)\right]\varphi(t)=0 \tag{14}$$

where

$$\omega^2(t)=\left(k_x^2+k_y^2-m^2\right)b^2t^2-k_z^2 \tag{15}$$

Hence, we obtain the oscillation range as

$$\sqrt{\left(k_x^2+k_y^2-m^2\right)b^2t^2-k_z^2}<\omega(t)<\left(k_x^2+k_y^2-m^2\right)b^2t^2-k_z^2 \tag{16}$$

5. Conclusion

In the present work, we obtained analytical solutions of the Klein-Gordon equation in the torsion gravity. The spacetime model is an anisotropic Bianchi-I type universe. Exact solutions are obtained in terms of Whittaker functions. By obtaining the oscillation frequency we present that scalar bosons have an oscillatory behavior in the specified universe. The oscillatory behavior of the scalar particles can be interpreted as possible scalar particle creation.

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