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HOBBIT SCALAR FIELD

M. SALTI, O. AYDOGDU, AND K. SOGUT

ABSTRACT

Plenty of scalar field proposals have been defined so far in order to answer the question "*Why the universe is expanding faster today than it did in its infancy?*". Although fundamental theories can help us to formulate different scalar field prescriptions, they do not define their self-interacting potentials in exact forms due to the complexity of corresponding equations. In the present work, we focus on the redefinition of the tachyonic scalar field dark energy description by making use of the Hobbit model. In Tolkien's "*The Lord of the Rings*" trilogy, the Hobbits resemble a mixture of three kinds of people mentioned in the book: the aspect of Men, the height of Dwarfs and pointed ears of Elfs. In the same way, the Hobbit model behaves as the three main fluids of the standard cosmology: dark matter, dark energy and dark radiation. Here, we use the Hobbit model for the reverse engineered tachyonic scalar field description.

1. Introduction

Recently, a plenty of astrophysical observations from different sources such as Supernova-Type Ia (SN-Ia henceforth)[1, 2], CMB anisotropies[3, 4], Large Scale Structures (LSS henceforth)[5, 6], Sloan Digital Sky Survey (SDSS henceforth)[7] and Planck-Results[8, 9, 10] have indicated that the universe is spatially flat at large scale and it has undergone two speedy expansion phases after the big bang: the first one happened before the radiation dominated epoch and the second one occurred not too long ago. At the present era, we are in this speedy enlargement period. In order to understand the speedy enlargement phase, an exotic type of constituent yclept dark energy (DE henceforth), which is dominating approximately 68.3 percent of the space-time tissue, is to be needed. Considering a dominant constituents which resemble familiar forms of matter or energy has not been justified yet due to they cannot be observed directly.

Diverse ideas have been given in literature to explain the accelerated enlargement epoch of the universe. The earliest and simplest idea is the famous cosmological constant[11]. After this pioneering model, various proposal to identify the DE have been introduced in literature: scalar fields[12, 13, 14, 15, 16],

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braneworld models[17, 18], assuming extra dimensions[19, 20, 21], modified gravity theories[22, 23, 24] and so on. Although these ideas can explain most of the astrophysical datasets, they fail to express the cosmic coincidence issue (*why the mysterious expansion is happening now and why it is speedy?*) and the fine-tuning puzzle (*why some cosmological parameters have exorbitantly high values while others do not take?*)[25]. Thus, to remove these problems, different dynamical DE proposals have been introduced in literature. The DE component of our universe is modelled generally by assuming a scalar field. The quintessence[26, 27] and tachyon scalar[28, 29, 30] field proposals have attracted lots of attention in modern cosmology. As we mentioned in the abstract, one can define different scalar field models via fundamental theories of physics such as the string/M theory, but they cannot help us to predict self-interacting potential of the corresponding scalar field model uniquely.

In this study, we consider an energy density prescription which is able to fit the current available data and leads to a speedy expansion era: the Hobbit model[31]. To reach this goal, it is assumed[31] that the space-time undergoes a radiation dominated era in the first step, then passes to a matter dominated phase and finally it enters to a de Sitter-like enlargement epoch with the energy density approaching a constant value asymptotically. Here, we compare the Hobbit energy density with the tachyon scalar field model in order to get an exact expression for the tachyonic self-interacting potential.

2. Preliminaries

The tachyonic model includes a very significant Equation-of-State (EoS henceforth) parameter, which is interpolating smoothly between 1 and 0[28]. Thus, the scalar field definition can be considered as a suitable candidate for the early time inflation phase as well as the late-time acceleration period[29, 30]. The tachyonic field is defined by the following effective Lagrangian density

$$\mathcal{L}_t = V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \tag{2.1}$$

where $V(\phi)$ represents the self-interaction potential while $g^{\mu\nu}$ describes the inverse metric tensor. For this model, the corresponding energy density and pressure are written, respectively, as[32]

$$\rho_t = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \qquad p_t = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{2.2}$$

Hence, one can easily find the EoS parameter of tachyonic field as

$$\omega_t = \frac{p_t}{\rho_t} = \dot{\phi}^2 - 1. \tag{2.3}$$

Note that the condition $-1 < \dot{\phi} < 1$ indicates a real tachyonic energy density otherwise we get an imaginary one. Subsequently, $-1 < \omega_t < 0$ is the corresponding constraint for the tachyonic EoS parameter which means the scalar field cannot behave like the phantom type DE.

In a general form, the Hobbit model is given by the following energy density expression[31]

$$\rho_h(a) = A\left[1 + \frac{s}{a}\right]^{\beta-\alpha}\left(1 + \left\{\frac{b}{a}\right\}^{\alpha}\right) \tag{2.4}$$

where $0 < \alpha < \beta$. Here, $a(t)$ is the cosmic scale factor, A represents a normalization constant and s and b (with $s < b$) denote scale parameters. After assuming the case $\alpha = 3$ and $\beta = 4$, the Hobbit model mimics a type of universe which is undergoing a radiation dominated phase in the first step, then a matter dominated era in the supsequent step and approaching a de Sitter type stage with constant energy in the final step[31].

3. The correspondence

Now, we are in a position to construct a correspondence between the Hobbit and tachyonic models.

As a preliminary step, we consider a flat Friedmann cosmological model of the universe filled with some perfect fluid

$$ds^2 = dt^2 - a^2(t)\sum_{i=1}^{3}(dx_i^2), \tag{3.1}$$

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - g_{\mu\nu}p, \tag{3.2}$$

where $\rho = \rho_b + \rho_h$, $p = p_h$, u_μ is the four-velocity vector and ρ_b represents ordinary (baryonic) pressureless matter. The Friedmann equation[33, 34], i.e. $H^2 = \frac{8\pi G}{3}\rho$ where $H = \frac{\dot{a}}{a}$ is the cosmic Hubble parameter, provides the dependence $\rho_h = \rho_h(t)$. Note that, here, the dot is denoting a derivative with respect to t and $\rho_h = \rho_m + \rho_e + \rho_r$ where the labels m, e and r denote pressureless dark matter (DM hencefoth), the DE and dark radiation (DR henceforth), respectively. It is important to mention here that, in further calculations, the present day values of cosmological quantities will be denoted by the subscript "0".

Next, the equation for energy conservation, i.e. $T_{\mu\nu;\nu} = 0$, yields the following results for a non-interacting case

$$\dot{\rho}_b = -3H\rho_b, \tag{3.3}$$

$$\dot{\rho}_h = -3H(\rho_h + p_h). \tag{3.4}$$

Therefore, the first relation given above indicates that $\rho_b = \rho_0^b a^{-3}$ and the second one fixes the pressure $p_h = p_h(t)$. So, making use of the obvious relation $\frac{d\rho_h}{dt} = \frac{d\rho_h}{da}\frac{da}{dt} = aH\frac{d\rho_h}{da}$, it is immediate to reach the below conclusion for pressure[31]

$$\begin{aligned} p_h &= -\frac{1}{3}\left[a\frac{d\rho_h}{da} + 3\rho_h\right] \\ &= A\left(1+\frac{s}{a}\right)^{\beta-\alpha}\left\{\frac{s(\beta-\alpha)}{3a\left(1+\frac{s}{a}\right)}\left[1+\left(\frac{b}{a}\right)^{\alpha}\right] + \left(\frac{\alpha}{3}-1\right)\left(\frac{b}{a}\right)^{\alpha} - 1\right\} \end{aligned} \tag{3.5}$$

which is indicating that the universe is not stationary.

On the other hand, it is known that, for cold dark matter (dust), $\omega_m = 0$. This basically implies that dust does not produce any pressure. Additionally, for radiation, we have $\omega_r = \frac{1}{3}$. Next, using the definition $\rho_h = \rho_m + \rho_e + \rho_r$, one can separate eqn.(3.4) into three different parts for a non-interacting case as given below

$$\dot{\rho}_m = -3H(\rho_m + p_m), \tag{3.6}$$

$$\dot{\rho}_r = -3H(\rho_r + p_r), \tag{3.7}$$

$$\dot{\rho}_e = -3H(\rho_e + p_e). \tag{3.8}$$

Now, assuming $\omega_m = 0$ and $\omega_r = \frac{1}{3}$, eqns. (3.7) and (3.8) can be rewritten in the following forms

$$\dot{\rho}_m + 3H\rho_m, \qquad \dot{\rho}_r = -4H\rho_r. \tag{3.9}$$

Thus, we get the following solutions for the DM and DR components

$$\rho_m = \rho_0^m a^{-3}, \qquad \rho_r = \rho_0^r a^{-4}. \tag{3.10}$$

Consequently, we figure out that

$$\begin{aligned} \rho_e &= \rho_h - \rho_m - \rho_r \\ &= A\left[1 + \frac{s}{a}\right]^{\beta-\alpha}\left(1 + \left\{\frac{b}{a}\right\}^{\alpha}\right) - \rho_0^m a^{-3} - \rho_0^m a^{-4}, \end{aligned} \tag{3.11}$$

$$\begin{aligned} p_e &= p_h - p_m - p_r \\ &= A\left(1 + \frac{s}{a}\right)^{\beta-\alpha}\left\{\frac{s(\beta-\alpha)}{3a\left(1+\frac{s}{a}\right)}\left[1 + \left(\frac{b}{a}\right)^{\alpha}\right] + \left(\frac{\alpha}{3} - 1\right)\left(\frac{b}{a}\right)^{\alpha} - 1\right\} \\ &\quad -\frac{\rho_0^r}{3}a^{-4}. \end{aligned} \tag{3.12}$$

As a result, for the DE component of the universe, after performing some algebra, one can get the following expression for the corresponding EoS parameter

$$\omega_e = \frac{A\left[1+\frac{s}{a}\right]^{\beta-\alpha}\left(1+\left\{\frac{b}{a}\right\}^{\alpha}\right) - \rho_0^m a^{-3} - \rho_0^r a^{-4}}{A\left(1+\frac{s}{a}\right)^{\beta-\alpha}\left\{\frac{s(\beta-\alpha)}{3a\left(1+\frac{s}{a}\right)}\left[1+\left(\frac{b}{a}\right)^{\alpha}\right] + \left(\frac{\alpha}{3}-1\right)\left(\frac{b}{a}\right)^{\alpha} - 1\right\} - \frac{\rho_0^r}{3}a^{-4}}. \tag{3.13}$$

In order to construct the correspondence between the Hobbit energy density and the tachyonic scaler field model, we identify that $\rho_t = \rho_e$, $p_t = p_e$ and $\omega_t = \omega_e$. From this point of view, it can be obtained that

$$V(\phi) = \rho_e(1 - \dot{\phi}^2)^{1/2}. \tag{3.14}$$

Then, making use of eqns. (2.3) and (3.13), we can write $\dot{\phi}^2 = 1 + \omega_e$ and get

$$\dot{\phi}^2 = 1 + \frac{A\left[1+\frac{s}{a}\right]^{\beta-\alpha}\left(1+\left\{\frac{b}{a}\right\}^{\alpha}\right) - \rho_0^m a^{-3} - \rho_0^r a^{-4}}{A\left(1+\frac{s}{a}\right)^{\beta-\alpha}\left\{\frac{s(\beta-\alpha)\left[1+\left(\frac{b}{a}\right)^{\alpha}\right]}{3a\left(1+\frac{s}{a}\right)} + \frac{\alpha-3}{3}\left(\frac{b}{a}\right)^{\alpha} - 1\right\} - \frac{\rho_0^r}{3}a^{-4}}. \tag{3.15}$$

Thence, the potential of the Hobbit tachyon field is written as

$$\begin{aligned} V(\phi) &= A\left[1+\frac{s}{a}\right]^{\beta-\alpha}\left(1+\left\{\frac{b}{a}\right\}^{\alpha}\right) - \rho_0^m a^{-3} - \rho_0^m a^{-4} \\ &\times \sqrt{\frac{\rho_0^m a^{-3} + \rho_0^r a^{-4} - A\left[1+\frac{s}{a}\right]^{\beta-\alpha}\left(1+\left\{\frac{b}{a}\right\}^{\alpha}\right)}{A\left(1+\frac{s}{a}\right)^{\beta-\alpha}\left\{\frac{s(\beta-\alpha)}{3a\left(1+\frac{s}{a}\right)}\left[1+\left(\frac{b}{a}\right)^{\alpha}\right] + \left(\frac{\alpha}{3}-1\right)\left(\frac{b}{a}\right)^{\alpha} - 1\right\} - \frac{\rho_0^r}{3}a^{-4}}} \end{aligned} \tag{3.16}$$

The expressions of kinetic term $\dot{\phi}^2$ and self-interacting potential $V(\phi)$ imply that such quantities may exist when $-1 \leq \omega_e \leq 0$. So, this indication shows that the phantom energy sector cannot be crossed in the selected cosmological scenario.

4. Conclusions and perspectives

The interesting "*Hobbit*" energy density model has recently been proposed to explain nature of the dark content dominated universe. Also, it is generally known that scalar field ideas of the DE can be assumed as an effective theory to investigate dark side of the universe. We believe that reconstruction of the scalar field prescriptions based on some energy density models may yield significant cosmological conclusions. Thus, this point motivated us to reformulate tachyonic scalar field model of the DE based on the Hobbit energy density proposal. It is significant to emphasize here that the concluded proposals with the redefined potentials are unique single-scalar ideas which can reproduce evolution of the universe.

In this study, we mainly established a connection between the tachyonic scalar field model of dark energy and the Hobbit energy density. Such calculations are very significant to understand how different dark energy ideas are mutually related to each other. Scalar fields have very attracting properties of explanting the phantom line crossing while the redefined potentials has meaningful cosmological conclusions.

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(M. Salti) Mersin University, Physics Department, 33343, Mersin, Turkey
E-mail address: msalti@mersin.edu.tr

(O. Aydogdu) Mersin University, Physics Department, 33343, Mersin, Turkey
E-mail address: oktaydogdu@mersin.edu.tr

(K. Sogut) Mersin University, Physics Department, 33343, Mersin, Turkey
E-mail address, e: kenansogut@mersin.edu.tr