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DYNAMICAL CASIMIR EFFECT IN THE ROBERTSON-WALKER SPACETIME

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ABSTRACT. Recent developments resulting from the modern colliders present the vacuum of the space is filled by the Higgs scalar field. Quantum Field Theory (QFT) determines infinite energy of the vacuum and suggests the vacuum of the space as essential source of the energy. The measure of the energy in a small amount of the vacuum can be modified by the substance surrounding it and this can lead dynamical Casimir effect. The dynamical Casimir effect, that total portion of the vacuum energy can be altered by placing two mirrors attracting each other, was interpreted by Schwinger to be the source of quantum particle creation. In the present study, we investigate scalar particle creation in Robertson-Walker Universe with a time-dependent scale factor $a(t) = \sqrt{\Gamma + \Lambda t}$. The considered problem is the dynamical Casimir effect. Exact solutions of the system will be obtained. Then, particle creation rate will be calculated by using Bogoulibov transformation technique.

1. INTRODUCTION

An attactive force occurs between two parallel conducting plates due to vacuum fluctuations. This phenomena is first predicted by Casimir at the end of the 50's [1]. The resultant forces and vacuum energies have static features. It is a very important phenomenon that could affect everything from micro-electronics to unified theories. It is supposed that the Universe has extra dimensions; 10- and 11-dimensional unified field theories of the fundamental forces are predicted by theorists. These dimensions are assumed to alter Newtonian gravitation at very "small" micrometre distances. For this reason determining the Casimir effect can provide to investigate the validity of such kind of extraordinary opinions. Casimir effect can be considered as the polarization of vacuum by boundary conditions. Alterations in the geometry by time can cause quantum particle creation and such process is called dynamical Casimir effect [2]. In the case of dynamic boundaries, this problem for conformally invariant fields in two-dimensional spacetime can be matched to the corresponding static problem and therefore provides an exact study. This effort is more complex in higher dimensions[3].

Particles spontaneously created from the vacuum is now assumed to be the reason of density fluctuations produced in inflation. Early attemps to understand this

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phenomena start with the study of Erwin Schrdinger in 1939 in which he has discussed the particle production by the space-time curvature[4]. Later, Parker has studied the particle creation by an external gravitational field[5]. Among the more recent studies, in 2001, Setare et.al. have been investigated the particle production from vacuum by expanding or contracting spherical shell with Dirichlet boundary conditions[6]. They have considered in another study also the case in which the sphere radius performs oscillation with a small amplitude and derived number of created particles to the first order of the perturbation theory[7].

In this study, we investigate particle creation process in radiation dominated RobertsonWalker space-time[8].

2. Scalar Particle Creation

Solving particle creation problem has many important physical results; it may enlighten the source of the great entropy in the present Universe. It requires to define vacuum "in" and "out" states, since the vacuum states are not unique in curved space-time. In the present case, we study a radiation dominated model of Universe given by the following metric:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{2.1}$$

where the scale factor $a(t) = \sqrt{\Gamma + \Lambda t}$ and Γ and Λ are constants.

Since the radius of curvature depends-on time, the case considered is a dynamical Casimir effect with moving boundaries[3]. By defining the conformal-time $d\eta = \frac{dt}{a(t)}$, the metric takes the following form

$$ds^2 = a^2(\eta)[d\eta^2 - (dx^2 + dy^2 + dz^2)] \tag{2.2}$$

The Klein-Gordon equation in curved space-time is[9]

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + (m^2 + \xi R)\phi = 0 \tag{2.3}$$

where $\xi = \dfrac{1}{6}$ is the dimensionless coupling constant and R scalar curvature

$$R = \frac{6(\ddot{a} + a)}{a^3} \tag{2.4}$$

where dot (.) represents the ordinary partial derivatives with respect to conformal-time η.

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + (m^2 + \xi R)\phi = 0 \tag{2.5}$$

By using the line element given in eq.(2.2), equation (2.3) reduces

$$\ddot{\phi} + \frac{2\dot{a}}{a}\dot{\phi} - \overrightarrow{\nabla}^2\phi + a^2[m^2 + 1 + \frac{\ddot{a}}{a}]\phi = 0 \tag{2.6}$$

By defining the wave function in the form $\phi(\eta, \overrightarrow{x}) = e^{i\overrightarrow{k}.\overrightarrow{x}}\varphi(\eta)a^{-\frac{3}{2}}(\eta)$, one gets

$$\left\{\partial_\delta^2 + \frac{\frac{1}{4} + \frac{(1-\overrightarrow{k}^2)}{\Lambda^2}}{\delta^2} + \frac{m^2}{\Lambda^2}\right\}\varphi(\delta) = 0 \tag{2.7}$$

where definition $\delta = e^{\eta\Lambda}$ is made. The solutions of this equation are Bessel functions[10] and given by

$$\varphi(\delta) = \sqrt{\delta} Z_\nu(\frac{m}{\Lambda}\delta) \tag{2.8}$$

with

$$\nu = \frac{\sqrt{\Lambda^2 + 4\overrightarrow{k} - 5}}{2\Lambda} \tag{2.9}$$

In order to discuss particle creation from vacuum we need to define positive and negative frequency modes.

Asymptotic behavior of Hankel functions for $u \to \infty$ [10],

$$H_\nu^{(1)}(u) \sim \sqrt{\frac{2}{\pi u}} e^{i(u - \frac{\nu\pi}{2} - \frac{\pi}{4})}, H_\nu^{(2)}(u) \sim \sqrt{\frac{2}{\pi u}} e^{-i(u - \frac{\nu\pi}{2} - \frac{\pi}{4})} \tag{2.10}$$

and the behavior of the Bessel functions for $u \to 0$ is

$$J_\nu(u) \sim \frac{u^\nu}{2^\nu \Gamma(\nu+1)} \tag{2.11}$$

Then, following the steps of Ref. [9, 11], we can write vacuum "in" and "out" solutions as below form, respectively

$$\varphi^+(\delta)_{\delta\to\infty} = \frac{A_\infty}{\delta} H_\nu^{(2)}(\frac{m}{\Lambda}\delta) \tag{2.12}$$

$$\varphi^+(\delta)_{\delta\to 0} = \frac{A_0}{\delta} J_\nu(\frac{m}{\Lambda}\delta) \tag{2.13}$$

$$\varphi^-(\delta)_{\delta\to 0} = \frac{A_0}{\delta} J_{-\nu}(\frac{m}{\Lambda}\delta) \tag{2.14}$$

The positive-frequency mode at $\delta \to \infty$ can be written as a linear combination of the positive and negative frequency modes at $\delta \to 0$ in the form:

$$\varphi_\infty^+(\delta) = \alpha\varphi_0^+(\delta) + \beta\varphi_0^-(\delta) \tag{2.15}$$

where α and β are Bogoliubov coefficients that relate vacuum "in" and "out" modes. The rate of their absolute square is found to be

$$\frac{|\alpha|^2}{|\beta|^2} = e^{-2\pi\tilde{\nu}} \tag{2.16}$$

where $\nu = i\tilde{\nu}$.

By using the normalization condition of the wave function (due to the Bose-Einstein statistics)$|\alpha|^2 - |\beta|^2 = 1$ and Eq. (2.16), the number density of the created scalar particles can be computed as follow

$$n \simeq |\beta|^2 = \frac{1}{[e^{-2\pi\tilde{\nu}} - 1]} \tag{2.17}$$

which has a thermal distribution form.

3. Conclusions

Vacuum fluctuations have detectable results which can be directly viewed in experiments on a microscopic scale, for example the passage of an excited atom to the its ground state by spontaneously emitting a photon is a result of vacuum fluctuations. Interaction with the dynamical boundaries affect the vacuum to create particles. In the present paper we deal with an another example of dynamical boundaries and calculate the particle creation from the time-dependent external gravitation fields. We study scalar fields that conformally coupled to a radiation dominant spacetime. Since the scale of the space depends on the time, we call the problem under consideration as the dynamical Casimir effect. We determine the particle creation number by using Bogoliubov coefficients.

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