ZDM Mathematics Education (2010) 42:483–491
DOI 10.1007/s11858-010-0258-5

ORIGINAL ARTICLE

Exploring the quality of the mathematical tasks in the new Turkish elementary school mathematics curriculum guidebook: the case of algebra

Behiye Ubuz · Ayhan Kürşat Erbaş · Bülent Çetinkaya · Meriç Özgeldi

Accepted: 22 April 2010 / Published online: 6 May 2010

Abstract To take its due place in the world of education, Turkey has been through serious reform initiatives in the curriculums of various school subjects since 2003. The new Turkish elementary school curriculum was prepared considering the research studies conducted in Turkey and in other countries, as well as the educational systems of developed countries and previous experiences with mathematics education in Turkey. This study attempts to provide a perspective on the nature of the instructional tasks in the new elementary school mathematics curriculum. In particular, our focus is to explore the level of cognitive demands (LCD) in the algebra tasks provided in the national elementary mathematics curriculum guidebook. This curriculum document is a major resource for administrators, stakeholders, textbook publishers and ultimately for teachers. For every learning objective, it provides sample tasks to be used in mathematics instructions. In this study, our purpose is to explore the LCD of each of these tasks by utilizing a framework developed by Smith and Stein (Math Teach Middle School 3:344–350, 1998). The framework classifies mathematical tasks according to the level of demands: lower-level and higher-level demands. While the lower-level demands are related to memorization and procedures without connections, the higher-level demands are related to procedures with connections and doing mathematics. The findings revealed that 60% of algebra tasks for each grade level required higher LCD and a great majority of the remaining tasks were at the level of procedures without connections. The findings of the study particularly inform curriculum developers about issues regarding the quality of the tasks given in the curriculum guide and provide possible suggestions to improve the implementation of the curriculum change process.

Keywords Cognitive demands · Algebra · Curriculum · Mathematical tasks

B. Ubuz (✉) · A. K. Erbaş · B. Çetinkaya
Department of Secondary Science and Mathematics Education, Middle East Technical University, 06531 Ankara, Turkey
e-mail: ubuz@metu.edu.tr

M. Özgeldi
Department of Elementary Education, Mersin University, Mersin, Turkey

1 Introduction

As a part of a longtime endeavor to join the European Union (EU), there have been several reform movements in Turkey to adapt to the EU standards and norms in social and political fields, including education (Erbas & Ulubay, 2008). These reforms should be perceived as Turkey's pursuit of attaining a competitive economy in the global market, achieving sustainable development and transforming into an information society. For example, for the generalization and development of basic education, the duration of compulsory education was increased from 5 to 8 years in 1997. Thus, the concept of middle grades (i.e., grades 6–8) was abandoned and elementary schools started to serve from grades 1–8 (see European Commission 2007 for an overview of the education system in Turkey). The extension of compulsory education, together with the strategic educational objectives of the EU, has considerable effects on curricular and instructional practices, particularly on the materials, activities and all other components of the learning environment. Furthermore, the poor mathematics performances of Turkish students in international comparative studies, such as the Repeat of Trends in International Mathematics and Science Study (TIMSS-R) and Programme for International Student Assessment (PISA), have alarmed stakeholders regarding the certainty of the existence of major problems in the mathematical education of these young students. Considering that Turkey has a centralized educational system and all students and teachers follow a national curriculum in mathematics (and other subject areas) at elementary (grades 1–8) and secondary (grades 9–12) levels, changes in the curriculums and approaches to what (school) mathematics is and how it should be taught have important consequences for future successes of Turkish students.

Recently, the Turkish Ministry of National Education (MoNE) has sought support to increase compulsory education to 12 years, as in many European and developed countries all over the world. On the other hand, initiated in 2003 by the Turkish Board of Education (Talim ve Terbiye Kurulu Başkanlığı-TTKB) and implemented gradually, the renovation of elementary and secondary education curriculums was a huge leap in terms of raising the quality of education in Turkey. For example, renovated programs in elementary mathematics for grades 6–8 were implemented gradually (starting from the 6th grade) beginning from 2006 to 2007 academic year with ongoing changes since then. Among others, renovation in mathematics curriculums for elementary education is of great interest to Turkish mathematics educators, policy makers, teachers, students and their parents (Erbas & Ulubay, 2008).

The vision of the new Turkish elementary school mathematics curriculum is based on a fundamental principle that "every child can learn mathematics" (TTKB, 2008, p. 7). Since mathematical concepts are generally abstract in nature, it is envisioned that teaching and learning of those concepts can be achieved using concrete and finite daily life models and using hands-on manipulatives. The new curriculum aims to raise individuals who: have independent thinking, decision-making and self-regulation competencies and skills; can solve mathematical problems and use mathematical ideas to solve real-life problems; communicate about and with mathematics; make connections among mathematical ideas and apply them in contexts outside of mathematics; reason within and with mathematics. There are emphases on an equal balance of conceptual and computational understanding in mathematics, using alternative assessment techniques and technology to teach and learn mathematics, and assisting students to develop positive attitude toward mathematics and motivation to learn it.

Theoretically, the curriculum is a complex construct with several facets consisting of goals, content, instruction, assessment and materials (Kilpatrick, 1996). To describe the content of the curriculum, different levels of curriculum are represented: the *intended*, the *implemented* and the *attained* curriculum (Robitaille et al., 1993). Specifically, the *intended* curriculum refers to the curriculum that exists at the educational system level and includes the aims and goals embodied in official documents (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). In the Turkish context, the official curriculum guidebooks prepared and published by the Board of Education (TTKB, 2008) portrays the intended elementary mathematics curriculum by providing in-depth background information about the philosophy, goals and approaches of the curriculum, content to be covered (with particular learning objectives for each content strand in each grade level) together with some sample introductory tasks and tips to be used in the classroom. Theoretically, curriculum guidebooks dictate what and how to be taught (and to what extent). Also, the textbooks prepared by either MoNE or private publishers are subject to review and approval by the Turkish Board of Education in line with specified evaluation criteria within the scope of provisions of the Regulation on the Review of the Instructional Materials before they can be used in schools as official textbooks. For elementary grades (i.e., grades 1–8), the textbooks are prepared in triple sets consisting of student edition, its workbook and the teacher edition. During the review process, the textbook is scored on the basis of a list of criteria prepared for each subject (e.g., mathematics) based on its curriculum guides. A textbook is expected to be reflective of the curriculum wholly.

This study investigated the intended curricular treatments of algebra in Turkish elementary school mathematics curriculum guidebook, particularly focusing on cognitive demands of the tasks. Cognitive demands mean "the kind and level of thinking required from students in order to successfully engage with and solve the task" (Stein, Smith, Henningsen, & Silver, 2000, p. 11). Therefore, the research reported in this article addressed the following research question: what is the nature of the treatment of algebra topics in elementary school mathematics curriculum guidebook across different grades? More specifically, what levels of cognitive demands (LCD) are required by tasks related to algebra, and what are the trends in the required LCD across grades?

The reason for delimitation to algebra was the perceived importance of algebra as an essential component of contemporary mathematics and its applications in many fields. Studying algebraic concepts is considered to provide a foundation for developing higher-order thinking and problem-solving abilities (NCTM, 2000). Without algebra, not only advancement into most areas of mathematics, such as analytic geometry, trigonometry, combinatorics, analysis and statistics, but also the study of other disciplines requiring mathematical abstraction and modeling such as science and engineering beyond the descriptive stage are limited if not impossible (Usiskin, 1995).

We believe that the examination of the LCD in the curriculum document enables teachers to engage their learning in choosing mathematical tasks for their teaching practice. Also, it is important that textbooks make possible a connection between the curriculum intentions and classroom activities. Therefore, the curriculum guidebook is a major resource for teachers and textbook writers during the implementation process of the new curriculum.

1.1 The emergence of algebra in school mathematics

Traditional introduction to algebra involves the study of variables, equations and expressions, moves toward progressively more abstract mathematical objects and relations and involves more complex techniques and representational forms (Carraher, Schliemann, & Schwartz, 2007; Kieran, 2007). On the other hand, instead of being an isolated course, Kaput (1998) argues that algebraic reasoning should be integrated across all grades and topics throughout the K-12 curriculum. In this context, early algebra (EA) movement intends to integrate algebraic reasoning into traditional topics of the elementary school curriculum and thus introduce algebra gradually to young learners (Carraher et al., 2007). Therefore, algebraic thinking for EA is employed as a bridge in algebra learning between elementary school students and older students at the secondary level (Kieran, 2006).

Algebraic thinking in the early grades requires developing ways of thinking, which include analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, providing and predicting (Kieran, 2004). In this developmental process, it is important that students should have opportunities to engage in the nature of processes, concepts and relationships within tasks (Stein et al., 2000). Moreover, these opportunities include the design of curriculums, which supports developing students' algebraic thinking through elementary school (Cai & Knuth, 2005). In line with this, algebra in Turkey is introduced at the upper elementary school (grades 6–8) as the extension of pattern recognition and investigations studied in lower elementary school, emphasizing the study of the relationships among a variety of representations and making connections among these representations. The goal is not to merely learn the structure and symbols of algebra, but to also use algebra as a tool to solve problems that arise in the real world. Thus, students' ability to generalize patterns and to show them using letters is the main skill, particularly at the upper elementary school. In the curriculum guidebook, increased importance is given to students doing mathematics, which means exploring mathematical ideas, solving problems, reasoning and communicating with other students. On the other hand, the mathematical tasks determine both what students learn and how they *think about, develop, use and make sense of mathematics* (Stein, Grover, & Henningsen, 1996, p. 459). Generally speaking, "tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information" (Doyle, 1983, p. 161). In this sense, tasks give students an opportunity to think conceptually, encourage them to make connections (Stein & Smith, 1998) and make them focus on a specific mathematical idea (Stein et al., 1996). Yet, researchers have tended to limit their attention to the examination of tasks either in mathematics textbooks or in standardized exams, with surprisingly little systematic investigation of curriculum guidebook documents. Therefore, examining the cognitive level of the tasks in curriculum documents may identify the nature of the mathematical tasks and their potential for students' learning.

1.2 A framework for analyzing mathematical tasks

Tasks, particularly academic ones, are defined as the products that students are expected to formulate and generate, and the resources available to students while they are generating the products (Doyle, 1983). From this perspective, curriculum developers design academic tasks to help students learn a skill or achieve insight into a particular situation (Stein & Kim, 2009). Consequently, mathematical tasks are designed according to "how students come to think about, develop, use, and make sense of mathematics" (Stein et al., 1996, p. 459).

There are various frameworks developed for analyzing structures including the cognitive dimensions of mathematical tasks [e.g., see the National Assessment of Educational Progress (NAEP), 2007; Mullis et al., 2005; OECD, 2006]. Among these, the LCD developed as a part of the project quantitative understanding: amplifying student achievement and reasoning (QUASAR) (Stein & Smith, 1998; Stein et al., 2000) has been used widely to analyze mathematical tasks in middle grades (Arbaugh & Brown, 2005; Jones & Tarr, 2007). LCD framework first distinguishes between mathematical tasks as lower-level and higher-level demands according to the LCD. While the lower-level demands are related to *memorization* and *procedures without connections*, the higher-level demands are related to *procedures with connections* and *doing mathematics*. Figure 1 presents each of these four levels with their distinguishing features.

LCD offers teachers a structure for examining mathematical tasks with respect to analyzing their potential, modifying them and using them (Ball, in Stein et al., 2000). Each of these elements is important to teachers in choosing and using tasks with students. Particularly, it can be worthwhile for teachers "to be more responsive to, and

Fig. 1 Characteristics of tasks at different levels of cognitive demand (Smith & Stein, 1998, p. 346)

Levels of Demands

Lower-level demands (Memorization):

• Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.

• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.

• Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.

• Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (Procedures without Connections):

• Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.

• Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.

• Have no connection to the concepts or meaning that underlie the procedure being used.

• Are focused on producing correct answers instead of on developing mathematical understanding.

• Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (Procedures with Connections):

• Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.

• Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.

• Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.

• Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (Doing Mathematics):

• Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.

• Require students to explore and understand the nature of mathematical concepts, processes, or relationships.

• Demand self-monitoring or self-regulation of one's own cognitive processes.

• Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

• Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.

• Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

supportive of, students' attempts to reason and make sense of mathematics" (Stein & Smith, 1998, p. 274). Arbaugh and Brown (2005) observed that mathematics teachers changed their patterns of task choice when they critically examined tasks using LCD. Their findings support that LCD is an appropriate framework to analyze mathematical tasks in middle school.

2 The method

2.1 Data sources

The study investigates the LCD required by tasks in the algebra strand through grades 6–8 in the elementary school mathematics curriculum guidebook (TTKB, 2008). We use the term algebra task (or simply task) to refer to an activity or set of questions in the curriculum that has been written with the intent of focusing students' attention on a particular idea in algebra. Any task that was provided under the algebra strand was considered as an algebra task. The algebra strand, like the other strands in the curriculum, follows a spiral approach developing the following sub-learning areas across grades 6–8: patterns and relations, algebraic expressions, equality, equations and inequality. To conduct an analysis of the algebra tasks with regard to the LCD, we examined all algebra tasks, including the activities and the questions asked in the explanation part. Therefore, the data source consisted of 72 tasks, including 43 activity examples and 29 questions. Activities or questions that build on one another are considered as a single task. In the curriculum guidebook while activities are suggested as mathematical tasks for explorations or investigations of mathematical topics by the students, questions are typical mathematical tasks such as exercises and problems. Teachers were expected to use the questions to supplement the activities recommended for use to introduce the main concepts and topics.

2.2 Analytical coding scheme and analysis

In this study, tasks were categorized using the task analysis framework developed by Smith and Stein (1998) (see Fig. 1). Each task was coded so that it was included into one of the following four categories: (1) task requires memorization (Low-M); (2) task requires procedures without connections (Low-P); (3) task requires procedures with connections (High-P); and (4) task requires doing mathematics (High-DM). The first three authors and another mathematics educator conducted the coding. The four raters met in a training session to discuss the sample coding done individually on the first five pages of the algebra strand for about 3 h before initiating data coding. In this training session, different opinions were further discussed and categorized on agreement. Following this training session, interrater agreement was computed between each rater. Reliability was calculated as the number of agreements divided by the number of agreements and disagreements and multiplied by 100. All reliability estimates were between 85 and 95%. All conflicts between the raters were resolved by consensus.

To clarify and discuss how we determined LCD of the tasks, we present examples of tasks that are categorized as Low-M and Low-P (see Fig. 2), as well as High-DM and High-P (see Fig. 3). In Fig. 2, Task 1 is categorized as Low-P because task instructions provide a procedure for plotting a figure on a coordinate plane and finding some of its coordinates. There exists only a little ambiguity about how to write the coordinates of the figure and complete the task (i.e., students only need to use their prior knowledge on procedures to write ordinate and abscissa for each ordered pair). Although the tasks ask for explanation, they simply require focusing on procedures that are used for completing the task. On the other hand, both questions in Task 2 are categorized as Low-M because they require students to reproduce previously learned facts about definitions of equality/inequality and algebraic expression. They neither require any algorithmic thinking nor involve any ambiguity.

Both tasks in Fig. 3 require higher LCD. While Task 3 is categorized at the level of High-DM, Task 4 is considered as an example of High-P. Task 3 is classified as High-DM because it requires students to explore and understand the relationship and make generalizations by identifying patterns. The task instructions do not suggest a pathway to be followed by the students, but ask them to think of and develop different solution strategies and use different representations, such as diagrams and manipulatives. Cognitive demand of Task 4 is also classified as higher-level High-P because it asks students to determine the sign of one variable depending on the sign of the other variable without suggesting an explicit pathway to the solution. This task can be solved by some degree of cognitive effort, which means students need to consider the meaning of their actions as they wrestle with the task.

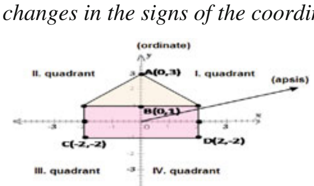
3 Results

The number of algebra tasks reviewed for this study was 72 altogether: 17, 20 and 35 for grades 6, 7 and 8, respectively. The number of tasks increases with the grade levels, while it is the least in the sixth grade in which formal introduction to algebra takes places.

Figure 4 portrays the distribution of LCD required by the algebra tasks. Within and across grade levels, the vast majority of the activities require high LCD, while the vast majority of questions require low LCD. While the nature of high-level tasks is predominantly procedures with connections in grades 6 and 7, it is both procedures with

Fig. 2 Sample tasks that require lower levels of cognitive demand [tasks were translated from the originals published in TTKB (2008, p. 293, 363)]

Task 1 (7th Grade): *Make figures (e.g., house, bird, etc.) by plotting points in the coordinate system. Have students write down the ordered pairs corresponding to these points and investigate the changes in the signs of the coordinates according to the quadrants they are.*



Task 2 (8th Grade): *Decide whether the propositions below are true or not. If wrong, correct it.*

1) *The expression "m > 4" shows an equality.*

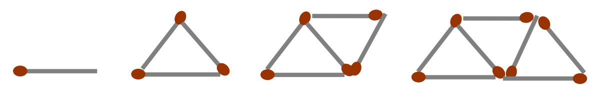
2) *9+12 is an algebraic expression.*

Fig. 3 Sample tasks that require higher levels of cognitive demand [tasks were translated from the originals published in TTKB (2008, p. 287, 364)]

Task 3 (7th Grade): *Have students choose a number pattern:* 1 3 5 7 …

A model that can be produced with various materials and a table both showing the number of materials used and explaining the relationship are presented below.

Model: *Have students model the patterns using a matchstick corresponding to number "1"*



1 **3** **5** **7** **…**

(1st number) (2nd number) (3rd number) (4th number) (nth number)

Table: *The relationship between a number and the number of matchsticks used*

The order of the number in the pattern	*The number of matchsticks used*	*The relationship between the number and the number of matchsticks used*		
		1st choice	*2nd choice*	*Other*
1	1	1+(1-1)=1	2·1-1=1	
2	3	2+(2-1)=3	2·2-1=3	
3	5	3+(3-1)=5	2·3-1=5	
4	7	4+(4-1)=7	2·4-1=7	
...	...	...	…	
n	**…**	**n+(n-1)**	**2n-1**	

Have students explain the relationship between the 1st and 2nd choices in the table verbally (e.g., the sum of the order no of the number in the pattern and its minus one, or twice the number no minus one

Task 4 (8th Grade): *Answer the following questions related to the inequality* $-2a + 1/2 < b$

- *If a is negative what is the sign of b?*
- *If b is negative, what is the sign of a?*

connections and doing mathematics in grade 8. The increase in the number of tasks in the category of doing mathematics is remarkable. Although there are only 2 tasks requiring doing mathematics in each of the grades 6 and 7, there are 12 in grade 8. Furthermore, there are only a few (1–2 or none) tasks requiring low LCD, memorization and procedures without connections in grades 6–8.

On the other hand, only about 40% of the tasks are low level, a majority of which (about 33%) require procedures without connections across all grade levels (see Fig. 4). Across three grades, tasks involving procedures without connections appear mostly in grade 7. Moreover, low-level tasks requiring only memorization appear mainly in grades 6 and 8 and there are none in the seventh grade. Of the high-level tasks in all grades, on the other hand, tasks requiring procedures with connections are twice the percentage of tasks requiring doing mathematics. Although about half of the tasks in grades 6 and 7 are procedures with connections, only about 30% of the tasks are of such types in grade 8. On the other hand, the highest-level tasks (i.e., doing mathematics) mostly appear in grade 8. While about 10% of the tasks require doing mathematics in each of the grades 6 and 7, about 30% of the tasks in grade 8 are such types.

4 Discussion and conclusion

Given that mathematics teachers are likely to rely on curriculum materials that are mainly influenced by the official mathematics curriculum guidebook (TTKB, 2008), this study examined the cognitive demands of grades 6–8 algebra tasks given in the guidebook. This study revealed a few important findings regarding the algebra tasks in the curriculum guidebook. Firstly, across the three grade levels, the mathematics curriculum guidebook comprises tasks at all the cognitive levels excluding the memorization level. Secondly, the distributions of higher and lower LCD

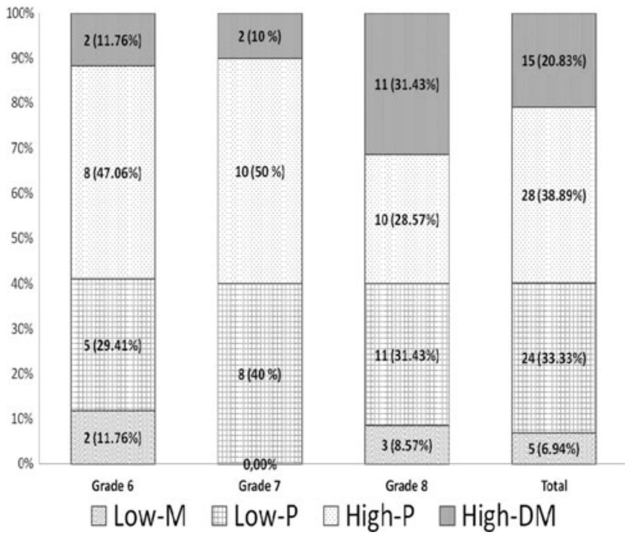


Fig. 4 Distribution of LCD required in tasks analyzed

of the algebra tasks across three grade levels show similar behavior. As aforementioned, about 60% of algebra tasks for each grade level require higher LCD and a great majority of the remaining tasks are at the level of procedures without connections. Thirdly, although the distributions of higher LCD are similar for grades 6 and 7, it is considerably different for grade 8. The percentage of tasks at the highest LCD (i.e., doing mathematics) for grade 8 is three times that of grades 6 and 7.

These findings show that the mathematics curriculum guidebook (TTKB, 2008), in general, encourages the development of student reasoning, complex and non-algorithmic thinking, and problem-solving skills. In this sense, the findings indicate that the range and levels of algebra tasks in the curriculum guidebook may well set an example for teachers, curriculum developers, and textbook writers who would like to appreciate the vision of new mathematics curriculum, which calls for the development of higher level of mathematical thinking and reasoning (TTKB, 2008). As mentioned by Jones and Tarr (2007), it is important to have tasks requiring higher LCD to provide students opportunities with experiencing “mathematics as more than a set of unrelated procedures and facts” (p. 20). This would eventually improve Turkish students’ poor performances in (mathematical) problem solving and low achievement in international studies such as TIMSS and PISA. As in the QUASAR project (Stein & Lane, 1996), students’ understanding and reasoning increase if teachers choose tasks requiring high cognitive demands and ensure that they maintain the demand of the tasks in the implementation process. That is, it is not sufficient to choose high-level tasks because the teachers need to maintain the demands of the task. Also, some classroom conditions may easily result in a decline in the demand of a high-level task. Furthermore, a teacher with effective teaching skills can easily increase the quality of a low-level task (Smith & Stein, 1998; Stein & Smith, 1998).

On the other hand, the distribution of the tasks indicates that the percentages of tasks at the level of procedures without connection are remarkable for all the grade levels (29% for grade 6, 40% for grade 7 and 31% for grade 8), and the percentages of tasks at the level of doing mathematics is the fewest among cognitive levels for the grades 6 and 7. One may argue that there should be fewer tasks at the level of procedures without connection and more tasks at the level of doing mathematics, since the tasks at low levels are believed to be used most frequently in a traditional classroom and teachers mainly need to see the sample of tasks at higher cognitive demands. However, although the number of tasks at the level of procedures without connection can be considered high, their inclusion in the mathematics curriculum guidebook is plausible. First of all, the tasks at this LCD, which generally require procedural skills, should not be seen as unnecessary or ineffective. The tasks ranging from practicing routine skills to developing conceptual understanding are important and should be included in curricular documents (Stein et al., 1996). Furthermore, the reasons for including tasks at the level of procedures without connection may have to do with the concerns of avoiding a wrong image that the new curriculum solely includes high-level tasks, focuses on only conceptual understanding and disregards procedural skills. It might also be the case that the authors of the guidebook intend to better assist users of the guidebook by presenting tasks and activities at different ranges and levels, so that they can have access to a sample of tasks with different LCD, which may give them vital information about the approaches and requirements of the new curricula.

It is also important to note the higher percentage of algebra tasks for grade 8 in the cognitive level of doing mathematics. Specifically, the algebraic tasks for the eighth grade level devote more attention to the transformational activities and tasks that require more abstract relations and complex processes. The significant increase in the percentage of algebra tasks in this cognitive level may reflect the intention of the developers of mathematics curriculum guidebook regarding the need of offering more higher-level tasks for the students, who are in the stage of making a transit to high school, where mathematical concepts begin to be taught at an increasingly abstract level. It can be concluded that the eighth grade exemplary algebraic tasks included in the mathematics curriculum guidebook are geared more toward developing algebraic thinking as they promote the development of ways of thinking as suggested by Kieran (2004).

The development of the mathematics curriculum guidebook for grades 6–8 (TTKB, 2008) is motivated on the assumption that it has the potential to shape the design and development of the new textbooks. Indeed, the curriculum guidebook in Turkey is regarded as a reference point for textbook developers, as the textbooks are eventually subject to review and approval on the basis of a list of criteria derived from the curriculum guide itself. On the criteria list, particular attention was directed to the search of higher-level cognitive tasks in the textbooks written. In this regard, it is reasonable to expect that new textbooks include an ample amount of algebraic tasks that require higher LCD. Future studies, therefore, may examine whether the new curriculum materials require higher LCD portrayed by the guidebook. Also, examination of the LCD required by algebra tasks in the textbooks that were published before and after the introduction of the new curriculum guidebook for grades 6–8 (TTKB, 2008) may reveal the trends on the LCD required in the textbooks and give us a clue about the impact of mathematics curriculum guidebook on this trend. The analysis of the implemented curriculum is beyond the scope of this study, but requires to be examined to determine the influence of the LCD on the implementation of the tasks. As mentioned by Arbaugh and Brown (2005), the level of tasks in the textbooks and how teachers implement them are also crucial. In essence, the use of Mathematical Tasks Framework (Smith & Stein, 1998) is not the only way of determining algebraic tasks; other frameworks or taxonomies for describing mathematical tasks exist, such as the Third International Mathematics and Science Study Assessment Framework (Mullis et al., 2005) or structure of the observed learning outcome (SOLO taxonomy, Biggs & Collis, 1982). We focused on the investigation of the LCD to use such a framework for critically determining algebra tasks in the mathematics curriculum guidebook for grades 6–8 (TTKB, 2008). Although this study was restricted to the algebra content, similar studies can be conducted to examine the distribution of the cognitive level of the tasks in other strands (i.e., geometry, number sense or measurement) in the curriculum guidebook. Comparing and contrasting the distribution of cognitive demands of the tasks in each strand of the guidebook may help in the revision efforts of the new curriculum.

References

Arbaugh, F., & Brown, C. A. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education, 8*, 499–536.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning—the SOLO taxonomy*. New York: Academic Press.

Cai, J., & Knuth, E. J. (2005). The development of students' algebraic thinking in earlier grades from curricular, instructional, and learning perspectives. *Zentralblatt fuer Didaktik der Mathematik (International Review on Mathematics Education), 37*(1), 1–4.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. (2007). Early algebra is not the same as algebra early. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 235–272). Mahwah, NJ: Erlbaum.

Doyle, W. (1983). Academic work. *Review of Educational Research, 53*, 159–199.

Erbas, A. K., & Ulubay, M. (2008). Implementation of the new Turkish primary education mathematics curriculum in the sixth grade: A survey of teachers' views. *The New Educational Review, 16*(3–4), 51–75.

European Commission. (2007). *Organization of the education system in Turkey (2008/09)*. Eurybase—The information database on education systems in Europe, Brussels. http://eacea.ec.europa.eu/education/eurydice/documents/eurybase/eurybase_full_reports/TR_EN.pdf. Accessed Dec 2009.

Jones, D. L., & Tarr, J. E. (2007). An examination of the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks. *Statistics Education Research Journal, 6*(2), 4–27.

Kaput, J. (1998). Transforming algebra from an engine of inequity to an engine of mathematical power by "algebrafying" the K-12 curriculum. In S. Fennel (Ed.), *The nature and role of algebra in the K-14 curriculum* (pp. 25–26). Washington, DC: National Academy Press.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator, 8*(1), 139–151.

Kieran, C. (2006). Research on the learning and teaching of algebra: A broadening of sources of meaning. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 11–49). Rotterdam: Sense Publishers.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. Lester (Ed.), *Handbook of research in mathematics education* (pp. 707–762). Greenwich: Information Age Publishing.

Kilpatrick, J. (1996). Introduction to section 1. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 4). Dordrecht: Kluwer.

Mullis, I. V. S., Martin, M. O., Ruddock, G. J., O'Sullivan, C. Y., Arora, A., & Erberber, E. (2005). *TIMSS 2007 assessment frameworks*. Chestnut Hill, MA: Boston College.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

OECD. (2006). *Assessing scientific, reading and mathematical literacy: A framework for PISA 2006*. Paris: OECD.

Robitaille, D. F., Schmidt, W. H., Raizen, S. A., McKnight, C. C., Britton, E. D., & Nicol, C. (1993). *Curriculum framework for mathematics and science* (Vol. TIMSS Monograph No.1). Vancouver: Pacific Educational Press.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: cross-national invention of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*, 344–350.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455–488.

Stein, M. K., & Kim, G. (2009). The role of mathematics curriculum materials in large-scale urban reform: An analysis of demands and opportunities for teacher learning. In J. Remillard, B. Herbel-Eisenmann, & G. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 37–55). New York: Routledge.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*, 50–80.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection. *Mathematics Teaching in the Middle School, 3*, 268–275.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Talim ve Terbiye Kurulu (TTKB). (2008). *İlköğretim matematik dersi 6–8 sınıflar öğretim programı ve kılavuzu [Curriculum and guidebook for elementary school mathematics course: grades 6 to 8].* Ankara: Milli Eğitim Bakanlığı. http://iogm.meb.gov.tr/pages.php?page=program. Accessed July 2008.

The National Assessment of Educational Progress (NAEP). (2007). *Mathematics framework for the 2007 national assessment of educational progress*. http://www.nagb.org. Accessed July 2008.

Usiskin, Z. (1995). Why is algebra important to learn? *American Educator, 19*(1), 30–37.