RAİNBOW CONNECTİON NUMBER OF SEQUENTİAL JOİNED GRAPHS

Özge COLAKOGLU[1]**,** Mehmet Ali BALCI [2], Pinar DUNDAR [3]

[1,2,3] Department of Mathematics, Faculty of Science, Ege University, Izmir, Turkey

ozgeeclkgl@gmail.com

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Abstract

Over the 150 years various works have been done on the coloring of graphs such as vertex coloring, edge coloring. The concept of rainbow coloring was introduced by Chartrand, John, McKeon and Zhang in 2008.[3]

Suppose that G represents a network (e.g., a cellular network). We wish to route messages between any two vertices in a pipeline, and require that each link on the route between the vertices (namely, each edge on the path) is assigned a distinct channel (e.g. a distinct frequency). Clearly, we want to minimize the number of distinct channels that we use in our network.

A path is rainbow if no two edges of it, are colored the same. An edge-coloring graph G is rainbow connected if any two vertices are connected by a rainbow path. An edge-coloring under which G is rainbow connected is called a rainbow coloring. Rainbow connection number, $rc(G)$, of a connected graph G is the minimum number of colors needed to color its edges, so that every pair of vertices is connected by at least one path in which no two edges are colored the same.

The operation on graphs are extensively studied in graph theory. In this paper, we study the rainbow connection number with respect to sequential join among the operations defined on graphs.

Keywords: Rainbow Coloring, Rainbow Connection Number, Sequential Joined graphs.

1. Introduction

Connectivity is perhaps the most fundamental graph theoretic property, both in the combinatorial sense and the algorithmic sense. There are many ways to strengthen the connectivity property, such as requiring hamiltonicity, k-connectivity, imposing bounds on the diameter, requiring the existence of edge-disjoint spanning trees, and so on.

A *graph* $G=(V,E)$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Connectivity asks for the minimum number of elements (nodes or edges) which need to be removed to disconnect the remaining nodes from each other. It is closely related to the theory of network flow problems. The connectivity of a graph is an important measure of its robustness as a network. A *cut*, vertex cut, or separating set of a connected graph G is a set of vertices whose removal renders G disconnected. The *connectivity* or *vertex connectivity* $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut.

Graph coloring is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color; this is called a *vertex coloring*. Similarly, an *edge coloring* assigns a color to each edge so that no two adjacent edges share the same color.

The *diameter* $diam(G)$ of a graph is the maximum eccentricity of any vertex in the graph. That is, it is the greatest distance between any pair of vertices. A *complete graph* is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge, denoted by Kn. A *star* S_k is the complete bipartite graph $K_{1,k}$, a tree with one internal node and k leaves.

An interesting way to quantitavely strengthen the connectivity requirement was recently introduction by Chartrand. An edge coloring of a graph is a function from its edge set to the set of natural number. A path in the graph is called a *rainbow path* if no two edges of the path are colored the same. An edge colored graph is called *rainbow connected* if every pair of vertices has a rainbow path between them. Such an edge coloring is called a *rainbow coloring* of the graph. Note that a rainbow coloring is possible only for connected graphs. The minimum number of colors required to rainbow color a connected graph, G, is called the *rainbow connection number* of G and is denoted by $rc(G)$.

For example it is easy to see that the rainbow connection number of a complete graph is 1 and that of a star graph is equal to the number of leaves. It is easy to see that in order to rainbow color a connected graph G it is enough to color all the edges a spanning tree of G using different colors. Hence the order of the graph minus one is a trivial upper bound for the rainbow connection number of the graph. The easy observation that a cycle with $k > 3$ vertices has rainbow connection $\lceil k/2 \rceil$. Also notice that, clearly, $rc(G) \geq diam(G)$ where $diam(G)$ denotes the diameter of G. If G is connected and has n vertices then $rc(G) \leq n-1$.

Sequential join is a graph operation that is introduced by Harary 1181. Obtained big graphs by using this operation, represent a communication network construction. In this work, we study the rainbow connection number with respect to sequential joined operation.

2. Rainbow Connection Number Of Sequential Joined Graphs

Definition 2. 1: Graphs G_1 and G_2 have disjoint node sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Their *union* $G = G_1 \cup G_2$ has, as expected, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. Their *join,* is denoted $G_1 + G_2$ and consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2.

Definition 2. 2: For three or more disjoint graphs $G_1, G_2, G_3, ..., G_{l-1}, G_l$, the *sequential join* $G_1 + G_2 + G_3 + ... + G_l$ is the graph $(G_1 + G_2) \cup (G_2 + G_3) \cup ... \cup (G_{l-1} + G_l)$.

Theorem 2. 1: Let G_1 be a nontrivial connected graph of order n and let G_2 be a nontrivial connected graph of order m. Then

$$rc(G_1 + G_2) = \begin{cases} 2 & m \leq 6 \text{ and } n \leq 6 \\ 3 & m > 6 \text{ or } n > 6 \end{cases}$$

Proof 2. 1:

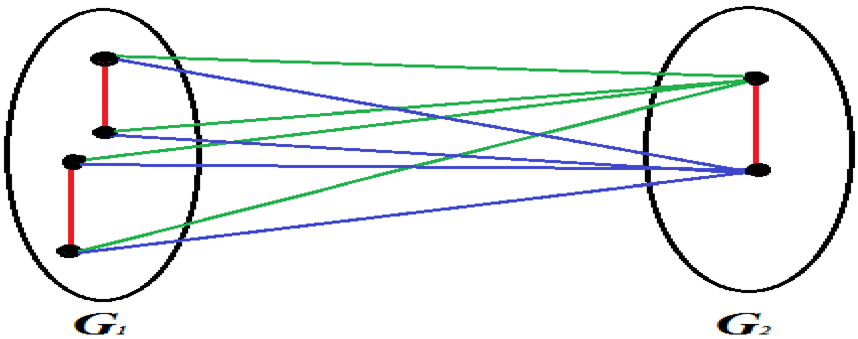

Figure 2. 1

From G_i ($i = 1, 2$) graph to G_j ($j = 1, 2$) graph is reached with one step. Therefore rainbow path has one color. From any vertex in G_i graph to any vertex in G_i graph can reach no more than 3 different color. Let $\{v_i, v_j\} \in V(G_1)$ and $\{v_k, v_{k+1}\} \in V(G_2)$. (v_i, v_k) edge must sign by a color, (v_k, v_{k+1}) edge must sign by second color and (v_{k+1}, v_j) edge must sign third color.

Theorem 2. 2: Let G_1, G_2 and G_3 be nontrivial connected graphs. n is order of G_1. m is order of G_2 and k is order of G_3. Then

$$rc((G_1 + G_2) \cup (G_2 + G_3)) = \begin{cases} 2 & n + m \le 6 \text{ and } k + n \le 6 \text{ and } m + k \le 6 \\ 3 & \text{otherwise} \end{cases}$$

Proof 2. 2:

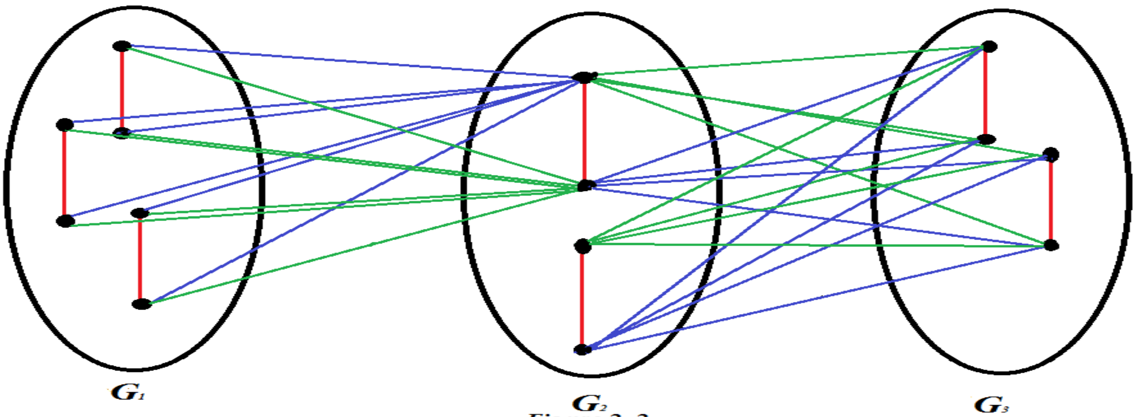

Figure 2. 2

Theorem 2. 3: Let $G = ((G_1 + G_2) \cup (G_2 + G_3) \cup ... \cup (G_{l-1} + G_l))$ $(l \ge 3)$ is connected. Then

$$rc(G) = l - 1 = diam(G)$$

Proof 2. 3:

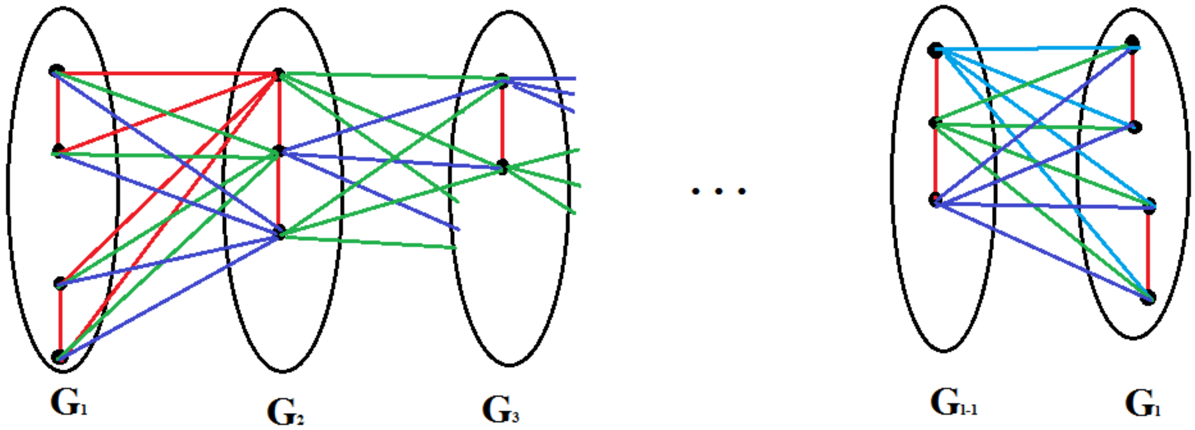

Figure 2. 3

According to theorem 2. 1 from G_i graph to G_i graph any of rainbow path are maximum three color. From any vertex in G_i graph to any vertex in G_j graph is reached by $|j-i|$ edge. Diameter is the greatest distance between any pair of verteces. Verteces in G_1 and G_l graphs has the greatest distance between pair verteces in G graph. From G_1 graph (or G_l graph) to G_l graph (or G_1 graph) is reached by $|l-1|$ edge. In that case, rainbow connection number of a G graph is diameter.

3. An Algorithm For Rainbow Connection Number Of Sequential Joined Graph

In this chapter, we give an algorithm to find rainbow connection number of sequential joined graph.

A0: Enter graphs and colors and $i = 0$

A1: Sequential join $G_1, \ldots, G_l$

A2: Paint same color all edge in G_i $(i = 1 \ldots l)$

A3: $i = i + 1$

A4: choose a vertex in G_{i+1} graph

A5: Paint first color joined edges, from choosen vertex in G_{i+1} graph to vertices in G_i graph, second color joined edges, from same vertex to vertices in G_{i+2} graph, for $i=1$.

A6: Paint different color joined edges, from choosen vertex in G_{i+1} graph to vertices in G_{i+2} graph.

A7: For the remaining edges in G graph, let v and u is adjacent vertex in G_{i+1} graph. Paint one color (different color from edges in G_i graph) joined edges, from v vertex to vertices in G_i graph. Paint second different color, from u vertex to vertices in G_i graph.

A8: $i < l$ then go to A3

A9: end.

Conclusions

In this study, rainbow coloring and rainbow connection number for sequential joined graphs are discussed. We used property join operation and rainbow coloring to evaluate rainbow connection number of sequential joined graphs.

References

[1] M. Basavaraju, L. Sunil Chandran, D. Rajendraprasad, A. Ramaswam; Rainbow connection number of Graph Power and Graph Product, arXiv:1104.4190v1 [math.CO] (2011)

[2] Fred Buckly, F. Harary; Distance in Graphs, New York(1989)

[3] G. Chartrand, G.L. Johns, K.A.Mc Keon, and P. Zhang, Rainbow Connection in Graph,(2008)

[4] G. Chartrand, P. Zhang: Chromatic Graph Theory; Chapman& Hall(2008)

[5] Kenneth H. Rosen; Discrete Mathematics and Its Applications; New York (2007)

[6] Xueliang Li, Yuefang Sun, Rainbow Connection of Graphs-Asurvey; arXiv:1101.5747v2 [math.CO] (2011)