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On the Volkenborn Integral of Some p-adic Trigonometric Function

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Abstract: Let p be a fixed odd prime number. By $\mathbb{Z}_p, \mathbb{Q}_p$ and $\mathbb{C}_p$ we denote the ring of p-adic integers, the field of p-adic numbers and the completion of the algebraic closure of $\mathbb{Q}_p$, respectively.

The Volkenborn integral of $f \in C^1(\mathbb{Z}_p \to \mathbb{C}_p)$ on $\mathbb{Z}_p$ is defined by the formula

$$\int_{\mathbb{Z}_p} f(x)dx := \lim_{n\to\infty} p^{-n} \sum_{j=0}^{p^n-1} f(j).$$

If $f(x) = c$ is a constant function, then

$$\int_{\mathbb{Z}_p} f(x)dx = c$$

and for any $f \in C^1(\mathbb{Z}_p \to K)$ the relation

$$\int_{\mathbb{Z}_p} f(x+1)dx - \int_{\mathbb{Z}_p} f(x)dx = f'(0)$$

holds (for detail see [3], [1], [2]) . We note that in $p-$adic analysis the elementary functions are defined by power series. For example, the exponential function is defined by

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

and it converges for $|x|_p < p^{-\frac{1}{p-1}}$.

In the present work we consider some p-adic trigonometric and inverse trigonemtric functions. We give some results on the Volkenborn integral of p-adic trigonometric and inverse trigonometric functions.

Keywords and phrases: p-adic numbers; p-adic trigonometric functions; Volkenborn integral.

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