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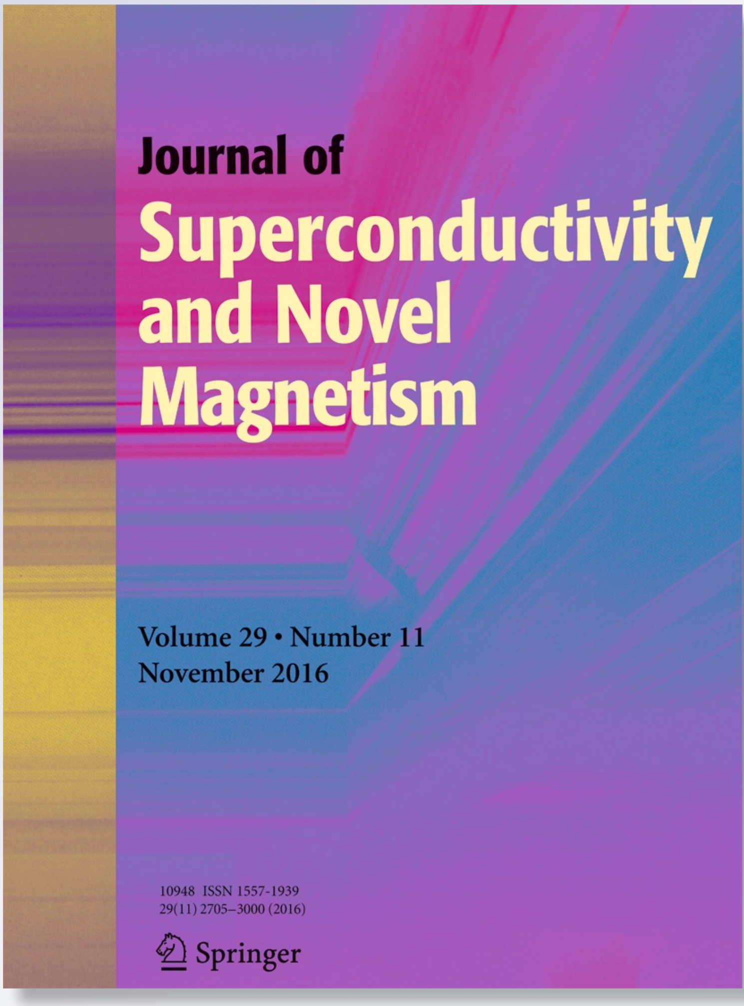

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ORIGINAL PAPER

Two-Dimensional Zigzag Domain Wall Structure in Ultrathin Films

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Abstract We investigate the zigzag domain wall structure in ultrathin magnetic films, where the easy direction of magnetization is perpendicular to the film. We calculate the dipolar energy as functions of the zigzag period and the film thickness for an equilibrium configuration in absence of external field. Provided that the domain wall size is large compared with the domain size, we find that the zigzag angle, θ, of zigzag domain wall decreases with the film thickness L. We discuss the results in connection with experimental data reported for amorphous TbCo films grown in external in plane magnetic field by high-frequency ion sputtering and for MnAs on GaAs films.

Keywords Ultrathin film · Domain walls · Domain structure

1 Introduction

The magnetic domain wall structures in ultrathin magnetic films have attracted much attention mainly due to aspects related to information storages. Magnetic domain walls in thin films are low-dimensional objects that separate regions with uniform magnetization [1].

Recent experiments [2, 3] have observed zigzag domain wall structure in amorphous rare-earth (RE)–transition metal (TM) films and MnAs-on-GaAs systems which are varied strongly with film thickness as discussed earlier workers experimentally [4–8] and theoretically [9–11]. The observed domain wall structures can arise from a balance between the exchange, dipolar, and anisotropy energies, as we propose in this study that the walls may have a two-dimensional structure.

This paper is concerned with films where the easy direction of magnetization is perpendicular to the film. The size of the zigzag domains is large, in the tens of microns size range, compared with the layer thickness of the order of 100–500 nm.

The aim of this study is to investigate the change in zigzag domain wall morphology and scale as a function of film thickness. We use a continuum model which is justifiable because the zigzag walls are so large compared with the lattice spacing. We explain the observed dependence of the zigzag domain wall size and the vertex angle on film thickness.

2 Calculation of Zigzag Morphology

As a starting point, the geometry of the film and its two-dimensional partition are shown in Fig. 1. The film is taken as an infinite slab parallel to the x–y plane with thickness L along the z coordinate. The magnetization in the antiparallel domains bounding the wall is constrained to be $+M_O$ for $x \leq -P$ and $-M_O$ for $x \geq +P$ in which P is the half-range period of the zigzag. The region is bounded by the thickness $z = \mp L/2$. The domain size, d, is taken to be much smaller than the zigzag wall width. The dipolar energy is found by expanding the magnetic potential in a Fourier series and matching to the boundary conditions as explained.

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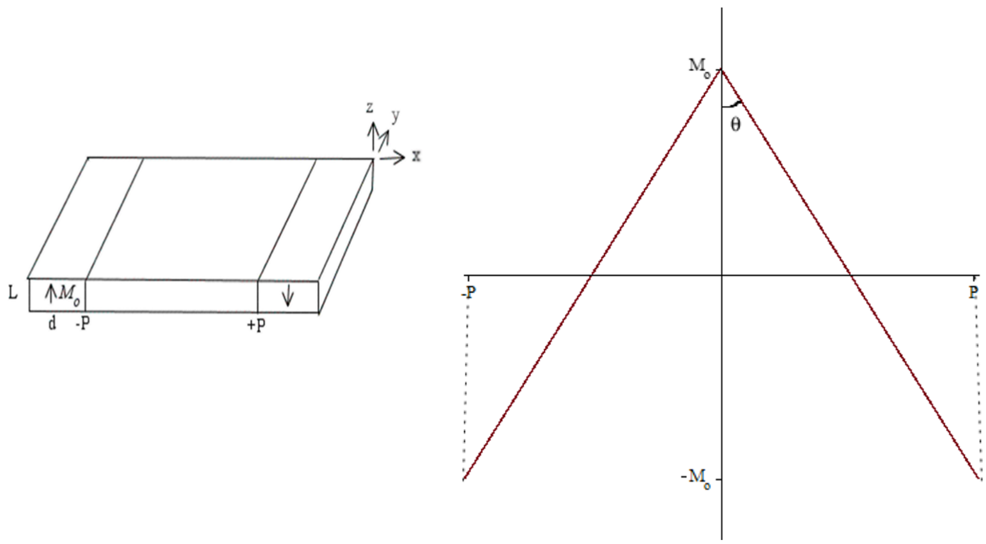
Fig. 1 Sketch of the parameters of the zigzag wall



We can obtained the dipolar energy for the zigzag pattern which is given in terms of a sum

$$E = \left(\frac{2}{\pi}\right)^5 \frac{2\pi M_o^2 L}{\delta} \sum_{\substack{n=1\\ \text{(odd)}}}^{\infty} \frac{1}{n^5}\left(1-e^{-n\pi\delta}\right) \tag{1}$$

where we define $\delta = {}^{L}/{}_{P}$ and the experimental value of δ is $\cong 10^{-3}$ for the films of interest [2]. We note that as the ratio ${}^{L}/{}_{P} \to 0$, the E tends to a single domain behaviour which is expected in very thin films [12]. However, the domain wall period is found by minimizing the dipolar energy with respect to P which is not analytic as ${}^{L}/{}_{P} \to 0$ so E must be evaluated with care. Equation (1) is valid for all δ. We now evaluate it for small δ. In (1), the summation approximates slowly to a convergence value. We split into a finite sum and an integral, here, N is chosen to be a finite value such that $N \gg 1$ but $N\delta \ll 1$ as follows:

$$\sum_{\substack{n=1\\ \text{(odd)}}}^{\infty} \frac{1}{n^5}\left(1-e^{-n\pi\delta}\right) \cong \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^5}\left(1-e^{-n\pi\delta}\right) + \frac{1}{2}\int_N^{\infty} \frac{dn}{n^5}\left(1-e^{-n\pi\delta}\right). \tag{2}$$

The result should be independent of N to leading order in δ. In this equation, the integral on the right-hand side (rhs) is well behaved and tabulated as follows [13]:

$$\int_N^{\infty} \frac{dn}{n^5}\left(1-e^{-n\pi\delta}\right) = \frac{\pi\delta}{3N^3} - \frac{\pi\delta^2}{4N^2} + \frac{\pi^3\delta^3}{6N} - \frac{25}{288}\pi^4\delta^4 + \frac{\gamma}{24}\pi^4\delta^4 + \frac{\pi^4\delta^4}{24}\ln(N\pi\delta) + O(\delta^5) \tag{3}$$

where $\gamma \approx 0.577$ is the Euler gamma constant. In this equation, the last term on the rhs is of order δ^5 and is neglected. Using the Taylor expansion and neglecting terms of order δ^5 (and higher), the summation part in (2) can be written as follows:

$$\sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^5}\left(1-e^{-n\pi\delta}\right) = \pi\delta \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^4} - \frac{\pi^2\delta^2}{2} \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^3} + \frac{\pi^3\delta^3}{6} \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^2} - \frac{\pi^4\delta^4}{24} \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n} + \ldots \tag{4}$$

In this case, the first term on the rhs becomes easily convergent to ${}^{\pi^4}/{}_{96}$ for large N. Thus, it can be written as follows:

$$\sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n^4} \cong \sum_{\substack{n=1\\ \text{(odd)}}}^{\infty} \frac{1}{n^4} - \frac{1}{2}\int_N^{\infty} \frac{dn}{n^4} \tag{5}$$

From this equation, we obtained $\left[\frac{\pi^4}{96} - \frac{1}{6N^3}\right]$. Similarly, the second and third terms on the rhs can be obtained $\left[\frac{7}{8}\xi(3) - \frac{1}{4N^2}\right]$ and $\left[\frac{\pi^2}{8} - \frac{1}{2N}\right]$ respectively and ξ is the Riemann zeta function with $\xi(3) \approx 1.202$. The last term on the rhs in (4) is the harmonic series and is divergent. We write the following:

$$S_N = \sum_{\substack{n=1\\ \text{(odd)}}}^{N} \frac{1}{n}$$

By considering $S_{N+2} - S_N$, we show that S_N can be written as follows for large N [14].

$$S_N = k + \frac{1}{2}\ln(N) + \frac{1}{2N} + O\left(\frac{1}{N^2}\right) \tag{6}$$

N is very large so we can neglect the terms of the order of $O\left({}^{1}/_{N^2}\right)$. The value of k was obtained by evaluating the series numerically and fitting to the above expression; we obtained $k = 0.635$ by using MAPLE program. Using these results in (4), we obtained the following:

$$\sum_{\substack{n=1\\(\text{odd})}}^{N} \frac{1}{n^5}\left(1-e^{-n\pi\delta}\right) = \frac{\pi^5\delta}{96} - \frac{\pi\delta}{6N^3} - \frac{\pi^2\delta^2}{2}\left(\frac{7}{8}\xi(3)\right) + \frac{\pi^2\delta^2}{8N^2} + \frac{\pi^5\delta^3}{48} - \frac{\pi^3\delta^3}{12N} - \frac{\pi^4\delta^4}{24}k - \frac{\pi^4\delta^4}{48}\ln(N). \qquad (7)$$

Substituting (3) and (7) in (2), one can see that the result is independent of N, so we write the dipolar energy of the zigzag pattern, ε:

$$\varepsilon = 64M_o^2 L\left(\tilde{b} + \tilde{c}\delta + \tilde{d}\delta^2 + \tilde{e}\delta^3 + \tilde{f}\delta^3 \ln\delta\right) \qquad (8)$$

where

$$\tilde{b} = \frac{\pi}{96}$$
$$\tilde{c} = -\frac{7\xi(3)}{16\pi^2}$$
$$\tilde{d} = \frac{\pi}{48}$$
$$\tilde{e} = -\frac{k}{24} - \frac{25}{256} + \frac{\gamma}{48} + \frac{ln\pi}{48}$$
$$\tilde{f} = \frac{1}{48}$$

For simplicity, taking into account the dipolar energy only, the following relationship for the zigzag period, P, was derived by minimization with respect to δ:

$$P = Le^{\frac{\text{Lambert}W(-\eta)}{\eta}} \qquad (9)$$

where $\eta = \frac{48}{\pi^4}(1-K) = -2.861$ with $K = \frac{\pi^5}{96} - \frac{7\pi^2\xi(3)}{8} + \frac{\pi^5}{16} + \left[-\frac{k}{6} - \frac{25}{144} + \frac{\gamma}{12} + \frac{ln\pi}{12}\right]\pi^4 + \frac{\pi^4}{12}$.

The fitting procedure correspondingly leads to the experimental values $\delta = 1.16$ and 1.35 for the thicknesses of 300- and 500-nm films respectively [3] and also 1.43 in our theory. For the sake of simplicity, we assume the dipolar energy only in this film. This simplification may be accurate quantitatively and seems to be adequate for qualitative analysis of the experimental data and our numerical result. For these parameters, the reduced dipolar energy versus the film thickness is shown in Fig. 2 in which the dipolar energy increases with increasing film thickness. Obviously, our theoretical and related experimental result are coincided. This indicates a good agreement between our theory and observed experimental results [3].

In Fig. 3, we represent the zigzag period as a function of the film thickness. As seen, there are three regions: (i) for $L \lesssim 100$ nm, the zigzag period, P, increases very fast with increasing thickness; (ii) for 100 $nm \lesssim L \lesssim 200$ nm, P increases dramatically with increasing L; and (iii) for $L \gtrsim 200$ nm, P increases very slowly with increasing L. Engel-Herbert et al. [3] reported that the 300-nm-thick film exhibits fully developed zigzag walls. This result supports our result in which above about 200 nm, the zigzag domains become stable with increasing film thickness.

Figure 4 shows the zigzag angle (2θ) as a function of the amplitude for three film thicknesses of 100, 300, and

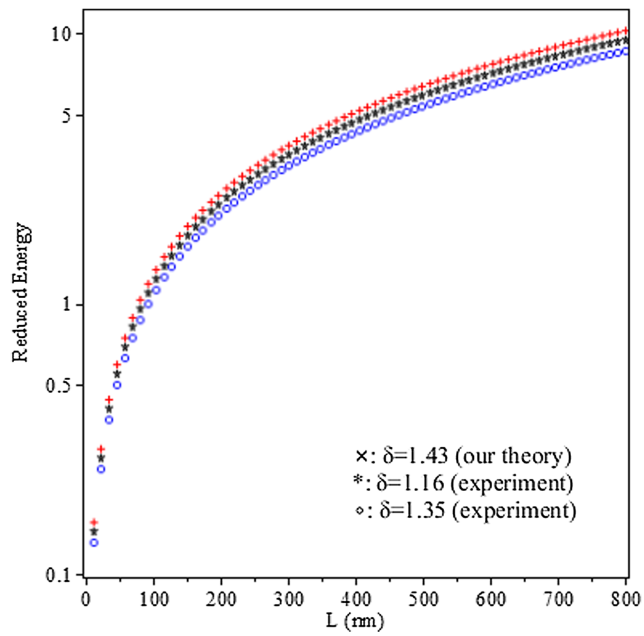


Fig. 2 Reduced dipolar energy as a function of the film thickness L

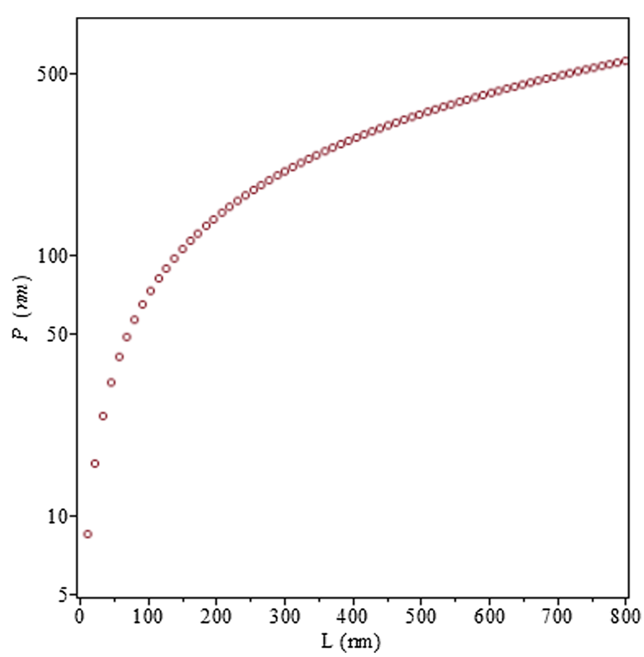


Fig. 3 The thickness dependence of the zigzag wall period

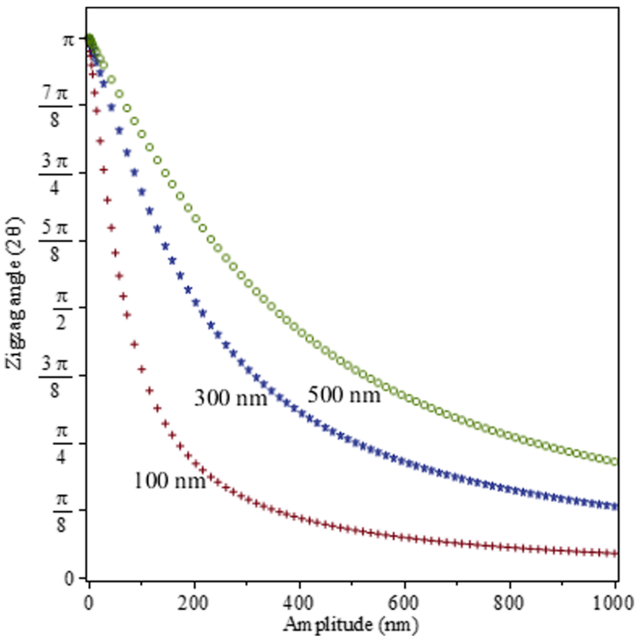


Fig. 4 The amplitude dependence of zigzag angle for films with various thicknesses

500 nm. As can be seen, the zigzag angle decreases with increasing amplitude. However, it increases with increasing film thicknesses. Ukleev et al. [2] have observed the zigzag pattern in some of their TbCo films with different thicknesses; however, they did not report any measured domain wall amplitude. And thus, we cannot perform a comparison between our calculation and their experimental results. Besides, Engel-Herbert et al. [3] observed the similar zigzag pattern in MnAs thin films with different thicknesses. They measured the mean values for the zigzag amplitude, the period and the vertex angle for two different film thicknesses of 300 nm (79°) and 500 nm (85°). By using their data for amplitudes of 367 nm (L = 300nm) and 464 nm (L = 500nm), we find the zigzag angles of about 60° and 74°, respectively. The minor difference between our and their results may be arised from the assumption of ignored energies of the exchange, wall and anisotropy.

In conclusion, we calculated the dipolar energy of the zigzag domain wall and compared our results with the recent experimental measurements of the characteristic parameters of a zigzag wall structure. We have shown that the dipolar energy increases with increasing film thickness.

The zigzag wall size increases with increasing film thickness and becomes more regular in shape. The vertex angle is inversely proportional to the zigzag amplitude; however, it increases with film thickness. It is interesting to remark that our results are very consistent with the measurements.

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