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Spatio-spectral analyses of electromagnetic wave energy absorption and heating effect

H.Z. Alisoy[a,*], B.B. Alagoz[a], S. Barlaz Us[a], F.N. Deniz[a], S. Alagoz[b]

[a] Inonu University, Department of Electric-Electronics Engineering, Malatya, Turkey
[b] Inonu University, Department of Physics, Malatya, Turkey

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ABSTRACT

This study presents a theoretical analysis method to calculate electromagnetic (EM) wave power absorption spectrum of materials by using attenuation coefficients. The heating effect of EM waves is modeled to analyze spectral distribution of temperature rises inside material body as a result of EM wave power absorption. These analyses are very useful for the investigation of electromagnetic wave-material interaction on the bases of electro-physical material parameters (permittivity, permeability and conductivity). An illustrative analysis of spatio-spectral distribution of EM wave energy absorption and resulting heating effect were conducted for muscle tissues and the results are discussed.

1. Introduction

Many technologies utilizing transmission properties of EM waves were developed the last century. These developments involves in many fields such as communication [1], medical technologies [2–4], heating processes [5–8], energy conversion [9]. Increasing demand in daily use of RF and microwave technologies in consumer electronics such as cellular phones, wireless communication raises questions and doubts about the negative effects of EM waves on public health [10–14]. Today, research studies on the effects of EM radiation on biological tissues are getting more and more important. Besides, use of EM waves for the propose of medical treatment has become one of substantial research topics of medical physics [2,4]. RF and microwave heating of foods is another beneficial application of EM waves [8].

Recent researches on interaction of EM waves with microstructured or composite materials promise novel developments for future world. Particularly, progress in metamaterials science [1,15–17] has been yielded novel applications such as photonic crystal perfect lens [18], waveguides [19]. Investigation and characterization of EM energy absorption properties of materials is very important on the way of practical implementation of large-scale microfabricated optoelectronic systems. Wave isolation features, heating of substrates and energy consumption of microfabricated systems are strongly depended of EM energy absorption properties of materials.

The penetration depth, attenuation of waves and temperature rise due to power absorption from EM wave varies from material to material, and choosing effective materials meeting application specifications is a necessity for technology development. Specifically, obtaining EM wave attenuation coefficient spectrum of materials is very beneficial for identifying penetration depth, energy absorption properties, isolation and heating features of the materials that are desired to utilize in EM technology development. This study is devoted to derive basic formulations to obtain attenuation coefficient spectrum depending on measurable electro-physical parameters of materials and suggests a spatio-spectral EM wave energy absorption and heating model. This model is based on energy conservation law postulating that energy difference between incoming and outgoing waves is absorbed by energy conservative systems. In this aspect, the spatial distribution of absorbed EM wave energy can be characterized by attenuation of EM waves while penetrating into the material.

The characterization of material properties associated with EM wave transmission is very substantial for the development of computer-aided EM technology development tools such as finite difference time domain (FDTD) wave simulation methods. FDTD based numerical simulations were effectively used to investigate transient properties of EM wave propagation [2–4,20,21]. However, there is still a need for analytical formulations to calculate spatio-spectral power absorption and heating properties of materials defined by measurable electro-physical material parameters: effective permittivity, permeability and conductance parameters.

* Corresponding author.
E-mail address: halis@inonu.edu.tr (H.Z. Alisoy).

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In the literature, Gorobets et al. presented an analytical solution for EM wave energy absorption throughout layers with smoothly varying dielectric parameters [22]. They considered absorbed energy at a cross-section of the material with non-uniform dielectric parameters as the remaining energy from reflected energy and transmitted energy from the material cross-section. In a recent work, Moritz successfully used EM wave penetration model based on exponential decay (Beer–Lambert Law) to obtain radial distribution of temperature in a thin lens due to absorption of light and heat conduction [7]. In our study, power dissipation due to the exponential decay of EM wave energy is assumed to turn into heat and increase the temperature of conservative materials with isotropic attenuation coefficient.

EM interaction with matters is depended on EM wave frequency. Because electro-physical and structural properties alter spectral response of matters. For instance, state of dipoles, free charge and atomic and molecular structure of substances affect EM wave propagation and absorption properties. Complexity of materials, as in biological materials, seriously complicates theoretical analyses, modeling efforts of the EM propagation and absorption mechanisms inside the material structures. The movement of bulk collection of charges is possible, which can be characterized by conductivity parameters. These charge motions resulting from EM interaction may cause the instant electric currents inside the materials or new EM radiations. A portion of EM wave energy spends for charges motions. Besides, vibration of molecular dipoles and atoms due to the varying fields of EM wave causes energy absorption and hence the heating of materials. In many recent works, EM energy absorption analyses were conducted by using specific absorption rate (SAR) on the bases of the conductivity of medium [2,11,21]. However, EM energy absorption of complex material depends on not only motion of free charges (space charges) and ions but also rotation of molecular dipole structures. Rotation and vibration states of molecules lead to an energy absorption mechanism so called dielectric heating, which is particularly effective in the microwave spectral region and infrared region. At lower frequencies, ion-drags due to EM waves are also a main factor in generation of thermal energy. Unless it radiates, all absorbed EM energy by means of these mechanisms turns into heat in the conservative system, and finally this results in a rise in body temperature of materials. Body temperature of materials is a measure of the average vibration energy of atoms and molecules composing the material. All these complicated and uncertain microscopic physical mechanisms involving in the EM waves-material interactions finds characterization in electro-physical parameters; permittivity, permeability and conductivity.

This study presents a theoretical model for the analysis of spatio-spectral distribution of EM wave power losses according to the exponential decay formulation (Beer–Lambert Law like) while propagating through a dispersive material. The attenuation coefficient of the material was expressed with respect to measurable electro-physical parameters of mediums (effective permittivity, effective permeability and effective conductance) and the EM wave frequency ($w = 2\pi f$). In this model, a decrease in energy of propagating EM waves in the material is assumed to be completely absorbed by dispersive materials exhibiting isotropic electro-physical parameters and the spatio-spectral energy absorption distribution and the spatio-spectral heating distribution are expressed analytically. The paper presents an illustrative analysis on muscle tissues characterized by permittivity, permeability and conductivity.

2. Methodology

Basic assumptions of the proposed energy absorption model are as follows:

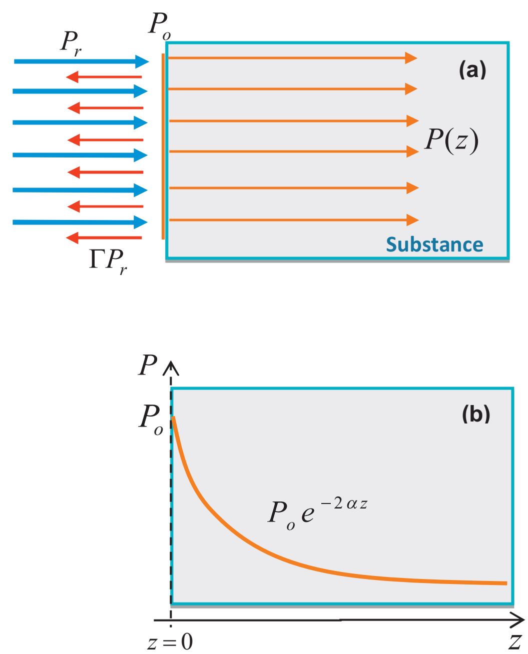

Fig. 1. (a) Ray-trace illustration of collimated EM waves propagating trough the substance; (b) an illustration for exponential decay of EM power in materials.

(i) Effective permittivity (ε), effective permeability (μ) and effective conductivity (σ) of materials are assumed to be invariant in time and space.
(ii) The whole energy lost by EM wave in materials is assumed to turns into heat in a conservative system.

The energy conveyed by electromagnetic wave are characterized by pointing vectors representing the directional energy flux density as [23]

$$\langle \vec{S} \rangle = \frac{1}{2} Re(\vec{E}_c \times \vec{H}_c^*) \quad (1)$$

The fields vectors in the complex form are expressed as,

$$\vec{E}_c = E_o \quad \exp(-\alpha z) \quad \exp(j(-\beta z + \psi) \times \vec{x}_0 \quad (2a)$$

$$\vec{H}_c^* = \left(\frac{1}{|Z_o|} \right) (E_o \quad \exp(-\alpha z) \quad \exp(j(\beta z - \psi) \times \vec{y}_0 \quad (2b)$$

where the parameter Ψ is phase difference between the field components $\vec{E}_c$ and $\vec{H}_c^*$. If the relation of $\vec{x}_0 \times \vec{y}_0 = \vec{z}_0$ is considered for the equal phases of $\vec{E}_c$ and $\vec{H}_c^*$, the average power is obtained as,

$$P = 0.5 \left(\frac{E_0^2}{|Z_0|} \right) \exp(-2\alpha z) = P_0 \quad \exp(-2\alpha z) \quad (3)$$

$$P_0 = \frac{1}{2} \frac{E_0^2}{|Z_0|} \quad (4)$$

where, $P_0 = P(z = 0^+)$ and $E_0 = E(z = 0^+)$ represent the average power and the electric field of EM waves penetrating into matter at the surface of material. The parameter Z_0 denotes characteristic impedance of materials. When incident EM wave comes from outside of the materials as in Fig. 1(a), EM wave penetrating into materials from the surface can be written with respect to the average power of incident waves (P_r) and the refraction coefficient (Γ) as $P_0 = P_r - \Gamma P_r$. Here, the refracted power from the surface $P(z = 0^-)$ is expressed by the term ΓP_r. Eq. (3) refers that energy conveyed by the electromagnetic waves exponentially decay depending on penetration depth z. The α parameter denotes the attenuation coefficient that is the imaginer part of propagation coefficient, defined as $\gamma = \beta + j\alpha$. Here, the parameter β is phase of propagation coefficient. In order to obtain dependence of α on the electro-physical parameters of materials (permittivity ε, permeability μ and the conductivity σ), the well-known Maxwell equations can be written as:

$$rot\vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t} = -jw\mu\mu_0\vec{H} \tag{5a}$$

$$rot\vec{H} = \sigma\vec{E} + \varepsilon_0\varepsilon_r \frac{\partial \vec{D}}{\partial t} = \sigma\vec{E} + jw\varepsilon_0\varepsilon_r\vec{E}. \tag{5a}$$

When Eq. (5a) is rearranged, one obtains,

$$rot\vec{H} = jw\vec{E}\left(\varepsilon_0\varepsilon_r - j\frac{\sigma}{w}\right) = jw\varepsilon_r'\vec{E} \tag{6}$$

where $\varepsilon_r' = \varepsilon_0\varepsilon_r - j\left(\sigma/w\right)$ is the complex permittivity of lossy materials. Besides, the wave number, $k = 2\pi/\lambda$, can be expressed as $k = w\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu}$ for lossless mediums. Accordingly, one can write the following equation,

$$(k')^2 = w^2\mu_0\mu \quad \varepsilon_r' = w^2\mu_0\mu \quad \varepsilon_0\varepsilon_r - j\sigma \quad w\mu_0\mu \tag{7}$$

In this case, the propagation coefficient $\gamma^2 = (k')^2$ is solved [23]; the attenuation coefficient α is obtained as,

$$\alpha(w) = \sqrt{\frac{w^2\varepsilon_0\varepsilon_r\mu_0\mu}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\varepsilon_0\varepsilon_r w}\right)^2} - 1\right)} \tag{8}$$

If EM wave velocity ($c = \sqrt{1/\varepsilon_0\mu_0}$) and the loss factor ($\tan\ \delta = \sigma/\varepsilon_r\varepsilon_0 w$) are considered, Eq. (8) is reorganized as,

$$\alpha(w) = \frac{w}{c}\sqrt{\frac{\varepsilon_r\mu}{2}\left(\sqrt{1 + (\tan\ \delta)^2} - 1\right)} \tag{9}$$

Eqs. (8) or (9) formulate attenuation coefficient depending on wave frequency and electro-physical material parameters – effective permittivity, effective permeability and effective conductivity. These formulas are used in Eq. (3) that models the exponential decaying character of wave power inside materials. Fig. 1(b) illustrates the spatial average power distribution of EM wave throughout a conservative material system.

Distribution of absorbed power is equal to the power lost by EM waves while traveling inside the materials. In this case, the power absorption density can be expressed as,

$$P_d(w, z) = -\frac{\partial P_{ort}}{\partial z} = 2\alpha(w)P_o e^{-2\alpha(w)z} \tag{10}$$

The total power absorbed by a Δz slice can be written as $P_s = P_d\Delta z$. If this internal power exchanged between EM waves and the material is fully converted into heat energy, the absorbed power leads to a temperature deviation, written as,

$$P_s = \frac{dQ}{dt} = C_h\rho\frac{dT}{dt} \tag{11}$$

Then, the spatio-spectral temperature raise in the material can be obtained as,

$$\Delta T(w, z) = \frac{2\alpha(w)\Delta t\Delta z}{C_h\rho}P_0 e^{-2\alpha(w)z} \tag{12}$$

where C_h is specific heat capacity, ρ is the material density and Δt is the EM wave exposure interval.

A consistent energy absorption model must comply with energy conservation law. In other words, the sum of spatial energy loss of penetrating waves must be equal to total energy gained by the material. The energy conservation relation can be written for EM power absorption of a conservative system as,

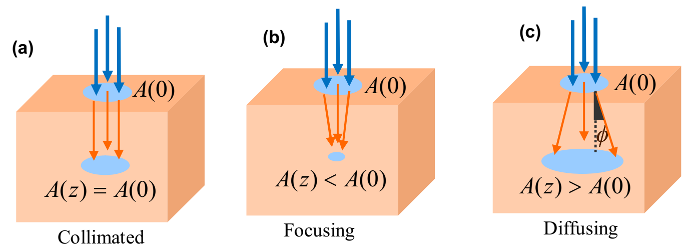

Fig. 2. (a) Ray-trace illustration of collimated EM waves propagating trough the substance; (b) Ray-trace illustration of focusing EM waves propagating trough the substance; (c) ray-trace illustration of diffusing EM waves propagating trough the substance.

$$P_o - P(w, z) = \int_0^z P_d(w, z)dz \tag{13}$$

Derivation of Eq. (13) can be found in Appendix section.

3. Effect of beam formations on EM power absorption

Fig. 2 illustrates volumetric power absorption models according to possible beam formations considering wave collimation, focusing and diffusion effects. The volumetric EM wave power absorption can be modeled by the sum of the power injected over material cross-sections, namely exposure cross-sections. Area of exposure cross-sections assumed to be in circular geometry is written by $A(z) = \pi\ \ r_z^2$, where r_z denotes radius of the circular cross section at the depth z. Considering areas of exposure cross-sections from the surface to a penetration depth, the following three cases of EM wave penetration are possible:

i. When $A(0)/A(z) = 1$, rays collimates as shown in Fig. 2(a).
ii. When $A(0)/A(z) > 1$, rays converge (focalize) as shown in Fig. 2(b).
iii. When $A(0)/A(z) < 1$, rays diverges (diffuse inside material) as shown in Fig. 2(c).

Here the beam formation described in the cases (i) and (ii) were shown for metamaterials at certain frequency bands [24–26]. Specifically, two dimensional photonic crystal slabs can exhibit directional self-collimation effect due to parallel group velocity vectors of waves at certain bands, or wave focalization effect as a result of negative effective refraction index. The wave diffusion (iii) occurs inside natural materials due to irregularly scattering of EM wave. Scattered EM wave from molecules or atoms travels inside the structure and forms a conical exposure volume inside material. The average power distribution (Eq. 3) considering effects of beam formations can be expressed as,

$$\vec{P}_A = \frac{A(0)}{A(z)}P_0 \quad \exp(-2\alpha z) \tag{14}$$

By considering EM wave diffusion in a circular exposure cross-section through the material, the diffusion attenuation term ($A(0)/A(z) < 1$) can be written as,

$$\frac{A(0)}{A(z)} = \frac{{r_0}^2}{(r_0 + z\tan\ \phi)^2} \quad \text{for} \quad \phi \geq 0 \tag{15}$$

where parameter ϕ is effective diffusion angle for the case of circular expansion of exposure cross-sections as in Fig. 2(c). The parameter r_0 denotes radius of the circular cross section at the surface ($z = 0$).

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Considering the diffusion of EM waves, the power absorbed by the substances can be expressed as,

$$P_d(w,z) = -\frac{\partial \vec{P}_A}{\partial z} = \xi P_o e^{-2\alpha(w)z}, \quad (16)$$

where the parameter ξ is obtained as follows,

$$\xi = \frac{2\tan\ \phi r_0^2}{(r_0 + z\tan\ \phi)^3} + \frac{2\alpha(w) r_0^2}{(r_0 + z\tan\ \phi)^2} = 2\frac{A(0)}{A(z)}\left(\frac{\tan\ \phi}{r_0 + z\tan\ \phi} + \alpha(w)\right) \quad \text{for} \quad \phi \geq 0 \quad (17)$$

Then, for EM wave diffusion in circular exposure cross-sections, the spatio-spectral temperature raise in the material can be obtained as,

$$\Delta T(w,z) = \frac{\xi \Delta t \Delta z}{C_h \rho} P_0 e^{-2\alpha(w)z} \quad (18)$$

4. An example for EM energy absorption and heating analysis of biological tissue

Increasing trend in daily use of wireless communication technologies makes the researches on EM waves-biological tissues interactions more important today. EM wave propagation in biological materials, energy absorption from EW waves and long-term and short-term effects of absorbed energy on the biological system were investigated in many works [11–14,21].

This section is devoted to demonstrate spatio-spectral analyses for EM energy absorption and EM heating of a muscle tissue model suggested by Sullivan [21]. Frequency dependence of effective $\varepsilon_r(w)$ and $\sigma(w)$ was written as,

$$\varepsilon_r(w) = \frac{\varepsilon_0}{\left(\varepsilon_\infty + \text{real}\left\{\varepsilon_s - \varepsilon_\infty/(1 + jw\tau_0)\right\}\right)} \quad (19)$$

$$(20)\sigma(w) = \sigma - \text{imag}\left\{\frac{\varepsilon_s - \varepsilon_\infty}{1 + jw\tau_0}\right\}$$

Between 40 to 433 MHz frequencies, electrical parameters of muscle tissue model were given as $\varepsilon_\infty = 15$, $\varepsilon_s = 120$, $\sigma = 0.64$ S/m, $\tau_0 = 6.6710^{-9}$ s [21]. In this example, we neglected the effect of wave diffusion inside the muscle in order to discuss results regardless of beam formations.

Fig. 3 shows the EM energy absorption and heating spectrum of muscle tissue calculated at a depth of 3.02 mm and a frequency ranges of 40–433 MHz. Fig. 3(a) presents EM frequency dependence of the attenuation coefficient (α). Higher frequencies cause more attenuation of EM waves and this results in more heating at the surface of muscle tissue. Fig. 3(b) clearly shows that EM wave loses more power in muscle tissue at the higher frequencies due to the high power absorption. Fig. 3(c) and (d) illustrates absorbed energy and the corresponding heating spectrum of the muscle tissue.

Fig. 4 reveals power of EM waves penetrating into muscle tissue at various frequencies. Fig. 4(a) and (b) indicates that the muscle tissues is more transparent for EM wave at low frequencies and this allows more power injection to the depths of tissue. The slope of logarithmic scaled EM wave power ($\log P$) in Fig. 4(b) gives mines two times attenuation coefficients of the material. In this sense; attenuation coefficients of materials can be obtained experimentally via the slope of logarithmic power transmission curve measured from material samples with different length.

Fig. 5 shows the spatial distribution of absorbed EM wave power throughout muscle tissue at various frequencies. Fig. 5(a) validates more power absorption at the surface for higher frequencies; however low frequency EM wave can convey more energy to deeper location. Logarithmic scaled EM wave power ($\log P_s$) in Fig. 5(b) demonstrates a transition region where absorbed power by the tissue are almost the same level for all frequencies in the range of

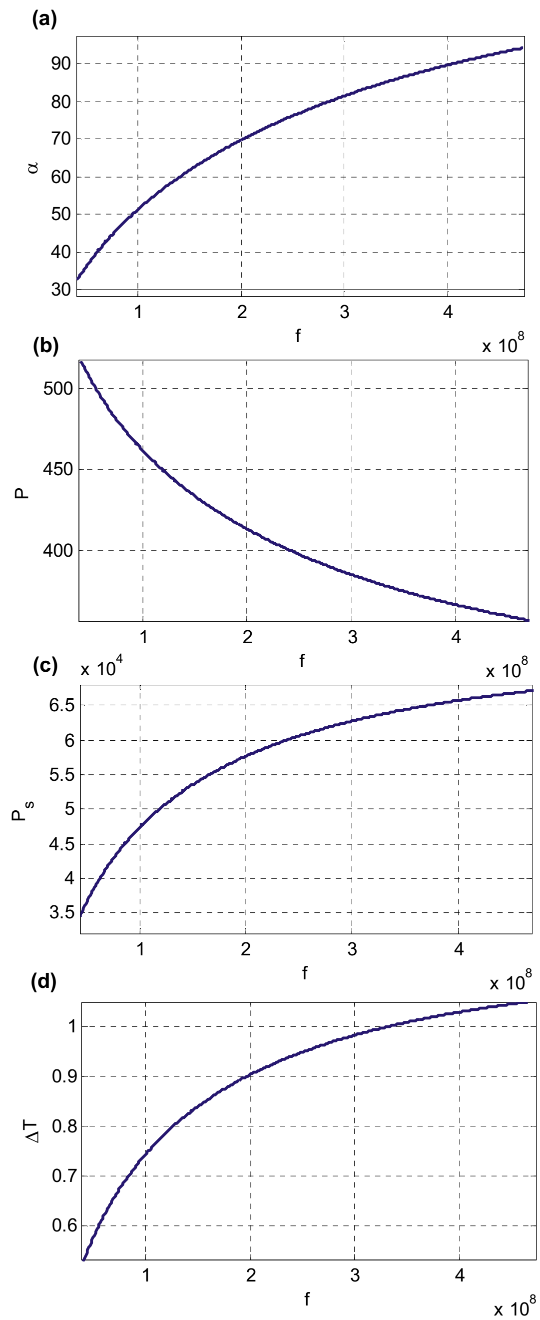

Fig. 3. α versus frequency (f [Hz]) plot in (a), P [W/m^2] versus frequency plot in (b), P_s [W/m^2] versus frequency plot in (c) and ΔT versus frequency plot in (d) for incident wave power $P_0 = 680$ W/m^2, depth of muscle $z = 3.02$ mm and application period $\Delta t = 60$ s.

40–433 MHz. Before the transition region, more power is absorbed at higher frequencies compared to low frequencies. This confirms that materials with high attenuation coefficient absorb more power around their surface and exhibits more temperature rises on the surface. Hence, muscle tissues can provide a better EM energy isolation as frequency of EM waves increases.

Fig. 6 shows the spatial distribution of temperature deviation due to absorbed EM wave power at various frequencies. Fig. 6(a) confirms more heating at the vicinity of surface for higher frequencies; however heating effect of low frequency EM waves is more effective in deep regions. Fig. 6(b) shows the logarithmic scaled temperature deviation characteristic ($\log \Delta T$). The transition region, where all heating curves are diverging, is also apparent in

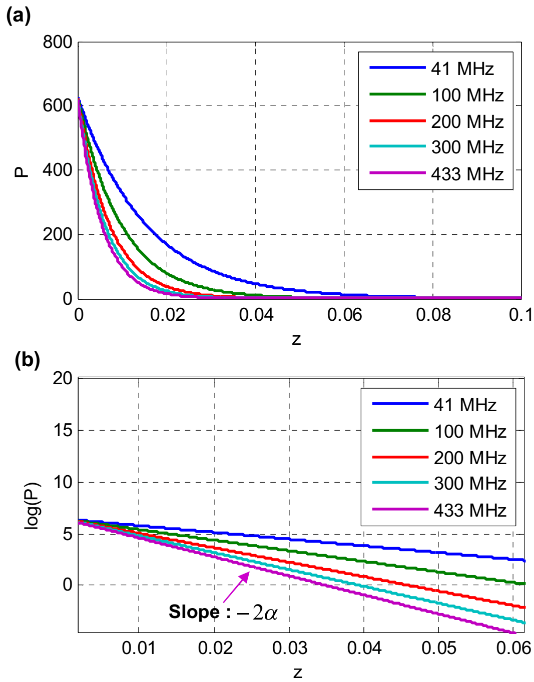

Fig. 4. (a) P [W/m^2] versus z[m] and (b) log P [W/m^2] versus z[m] for the incident wave power P_0 = 680 W/m^2 and various EM frequencies.

Fig. 6(b). According to Eq. (12), heating on the material surface can be calculated by the formula of $\Delta T(w, 0) = V_T \Delta t$, where the heating velocity is $V_T = 2\alpha(w)\Delta_Z P_0/(C_h\rho)$. As demonstrated in Fig. 6, the degree of temperature rise near the surface depends on attenuation coefficient and hence the frequency of EM waves. Besides, specific heat capacity (C_h) and material density (ρ) are parameters slowing down temperature rise on the surface during an application interval (Δt). The power of incident wave (P_r) is a factor directly increasing the surface heat because of $P_0 = P_r - \Gamma P_r$.

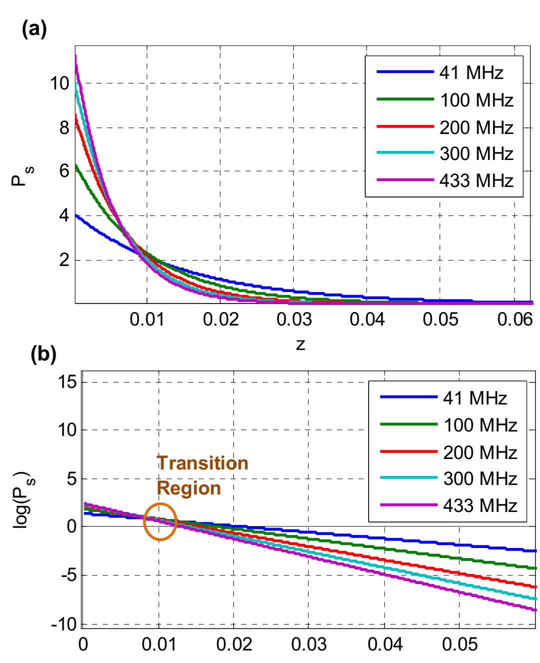

Fig. 5. (a) P_s [W/m^2] versus z[m] and (b) log P_s [W/m^2] versus z [m] for incident wave power P_0 = 680 W/m^2 and various EM frequencies.

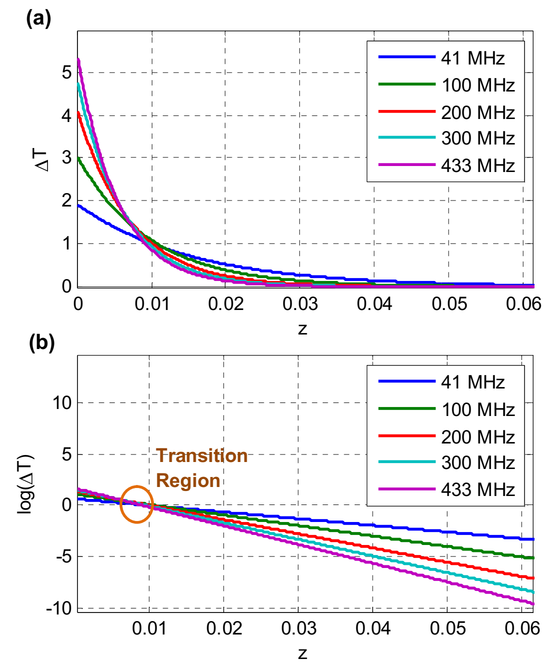

Fig. 6. (a) ΔT [C^0] versus z[m] and (b) $\log_{10} \Delta T$ [C^0] versus z [W/m^2] for incident wave power P_0 = 680 W/m^2 and the exposure time Δt = 30 s.

Fig. 7 shows the spatio-spectral distribution of logarithmic scaled power absorption characteristic and heating calculated for muscle tissues. Fig. 7(a) confirms a more EM wave energy absorption throughout the material at lower frequencies. As the frequency increases, energy of EM waves is largely absorbed near the surface

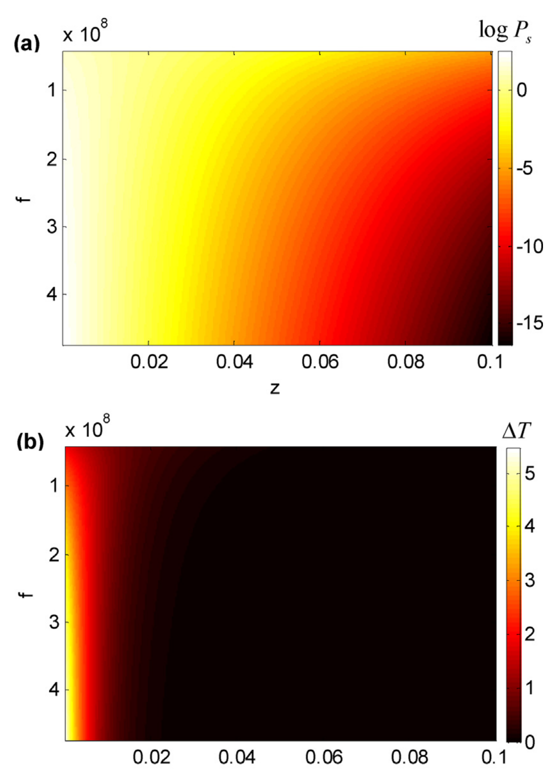

Fig. 7. (a) log P_s [W/m^2] versus z [m] –f [Hz] for incident wave power P_0 = 680 W/m^2 and (b) ΔT [C^0] versus z[m] –f [Hz] for incident wave power P_0 = 680 W/m^2 and the exposure time Δt = 30 s.

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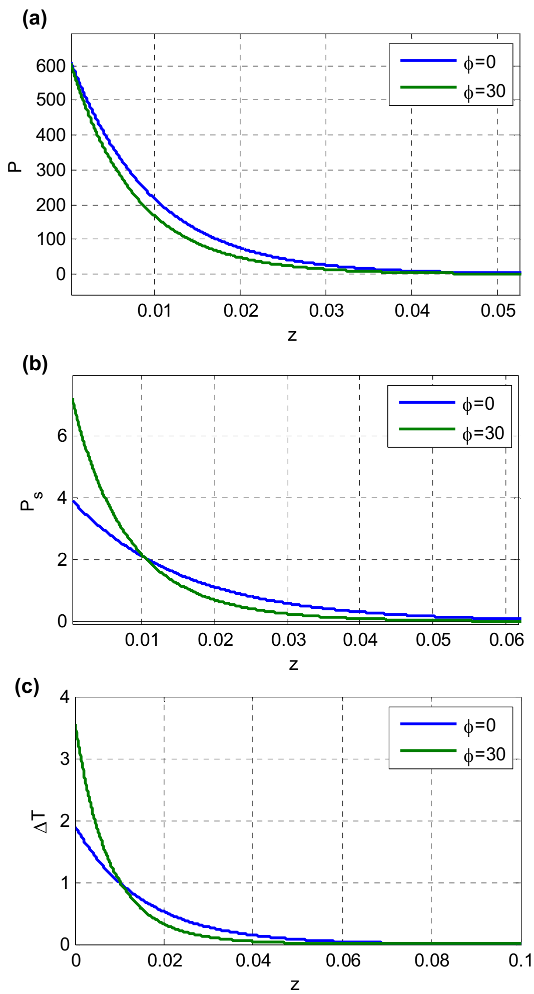

Fig. 8. (a) A comparison of spatial distribution of P [W/m^2] characteristics for collimated wave propagation ($\phi = 0$) and diffusing wave propagation with $\phi = 30$. (b) A comparison of P_s [W/m^2] characteristics for collimated wave propagation ($\phi = 0$) and diffusing wave propagation with $\phi = 30$.

due to high attenuation. Fig. 7(b) shows spatial-spectral distribution of EM wave heating of muscles.

The energy absorption calculation obeys energy conservation relation given by Eq. (13). As a result of rounding errors originating from use of a finite digit length in computations and error coming from discrete integration technique, a deviation from the conservation relation at a level between 8.5×10^{-2}% and 4.8×10^{-2}% of $P_0 = 680$ W/m^2 was observed between 40–433 MHz frequencies for a discrete integration step length dz = 0.00001. This accuracy is suitable for reliable analyses. As the step length is decreased, the calculation results become more consistent with energy conservation relation defined by Eq. (13).

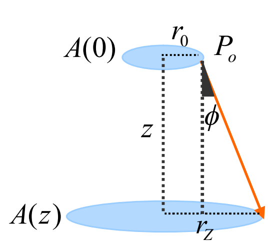

Fig. 9. A geometrical analysis for diffusing of wave in the matter.

Fig. 8(a)–(c) compares P, P_s and ΔT characteristics for collimated wave propagation ($\phi = 0$) and diffusing wave propagation with $\phi = 30$. Diffusion of waves into larger areas of the tissue causes more power absorption at the surface because it becomes a factor strengthening wave attenuation effect. Therefore, wave diffusion leads to more heating at the surface than collimated wave penetration. However, collimated waves can convey more power to deeper regions.

5. Conclusions

This study presents a theoretical model to obtain the power absorption and heating spectrum of a material layer exposed EM waves. The model is based on exponential decaying of EM waves inside the material with respect to attenuation coefficients, and this model well complies with energy conservation law. Attenuation coefficient is expressed as a function of wave frequency and measurable electro-physical parameters of materials such as effective permittivity, effective permeability and effective conductivity.

The model allows calculating spatio-spectral distribution of power absorbed from EM waves and resulting heating effect. These distributions are very useful to characterize EM wave absorption properties of materials and this is very desirable when deciding the optimal materials for implementation of EM wave technologies. Especially, a spatio-spectral EM energy absorption and heating characterization of biological material are very beneficial for the investigation of EM wave-tissue interactions. Analyses conducted on muscle tissue demonstrated that higher frequency in RF cannot efficiently heat deep muscle regions. The most of EM energy is absorbed around surfaces and thus increases surface temperature effectively.

Appendix A.

Derivation of Eq. (13):

According to energy conservation law, energy loosed by EM waves (E_L) should be equal to energy gained by the material (E_G). This can be simply expressed as,

$$E_L - E_G = 0 \tag{21}$$

One can write the energy lost by EM wave at the depth z as

$$E_L = P_o \Delta t - P(w, z) \Delta t \tag{22}$$

where the term of $P_o \Delta t$ is wave energy at the surface of material and the term of $P(w,z)\Delta t$ stands for energy of penetrating waves at the depth z inside the material.

Gained energy by the material body can be written as the sum of spatial energy absorptions throughout the material as,

$$E_G = \int_0^z P_d(w, z) \Delta t \ \ \mathrm{d}z. \tag{23}$$

When Eq. (21) is reorganized by using Eqs. (22) and (23), energy conservation relation for the power absorbing conservative system can be obtained as $P_o - P(w, z) = \int_0^z P_d(w, z) \mathrm{d}z$.

Derivation of Eq. (15):

Considering geometrical analysis given in Fig. 9, one can write for the radius r_z as

$$r_z = r_0 + z \tan \ \phi \tag{24}$$

Assuming to the circular form of exposure cross sections, the ratio of the exposure cross sections at the surface, $A(0) = \pi \ \ r_0^2$, and the exposure cross section at the depth z, $A(z) = \pi \ r_z^2$, can be written as, $A(0)/A(z) = (r_0/(r_0 + z\tan \ \phi))^2$.

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