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On Corona Product of Two Fuzzy Graphs

Özge Çolakoğlu Havare **and** ***Hamza Menken***

Department of Mathematics, Science and Arts Faculty, Mersin University
Mersin, Turkey, 33343
E-mail: ozgecolakoglu@mersin.edu.tr; hmenken@mersin.edu.tr

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Abstract. Corona product of two fuzzy graphs which is analogous to the concept corona product operation in crisp graph theory is defined. The degree of an edge in corona product of fuzzy graphs is obtained. Also, the degree of an edge in fuzzy graph formed by this operation in terms of the degree of edges in the given fuzzy graphs in some particular cases is found. Moreover, it is proved that corona product of two fuzzy graphs is effective when two fuzzy graphs is effective fuzzy graphs.

Keywords: Corona product; degree of an edge; effective fuzzy graph

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

It was Rosenfeld who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975 [10]. Later on, Bhattacharya gave some remarks on fuzzy graphs [1]. The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Moderson and Peng [4]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations were discussed by Nagoorgani and Radha [6]. Radha and Kumaravel introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs [8] and studied about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of union and join [9].

In this paper, we have introduced the concept of corona product of fuzzy graphs, which are analogous to the concept corona product in crisp graph theory. We study about the degree of an edge in fuzzy graph which are obtained from two fuzzy graphs using corona product operation. The degree of an edge in the corona product of two fuzzy graphs is obtained in some particular case. Moreover, it is proved that corona product of two effective fuzzy graphs is an effective fuzzy graph.

Let V be a nonempty set. A fuzzy graph is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of v and μ is a symmetric fuzzy relation on $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ such that $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$ for all u,v in V [5]. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(\mathrm{V},\mathrm{E})$ where $E\subseteq V\times V$. $\mu(u,v)>0$ for $(\mathrm{u},\mathrm{v})\in E$, $\mu(u,v)=0$ for $(u,v)\notin E$.

Throughout this paper we assume that μ is reflexive and need not consider loops. Note that $G_i : (\sigma_i, \mu_i)$ denote fuzzy graphs with underlying crisp graphs $G^*_i : (\mathrm{V}_i, \mathrm{E}_i),\quad \mathrm{i} = 1,2$ with $|V_i| = p_i,\quad i = 1,2$. Also, the underlying set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graph in all the examples and all these properties are satisfied for all fuzzy graphs except null graphs. We shall denote the edge between two vertices u and v by uv.

In [6], the degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv) \tag{1.1}$$

By Nagoorgani and Ahamed in [8], the order of a fuzzy graph G is defined by

$$O(G) = \sum_{u \in V} \sigma(\mathrm{u}) \cdot \tag{1.2}$$

The union of two fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ is defined as a fuzzy graph $G = G_1 \cup \mathrm{G}_2 : (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ on $G^* : (\mathrm{V}, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$ with

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u), & u \in V_1 - V_2 \\ \sigma_2(u), & u \in V_2 - V_1 \\ \sigma_1(u) \vee \sigma_2(u), & u \in V_1 \cap V_2 \end{cases}$$

and

$$(\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(uv), & uv \in E_1 - E_2 \\ \mu_2(uv), & uv \in E_2 - E_1 \\ \mu_1(uv) \vee \mu_2(uv), & uv \in E_1 \cap E_2 \end{cases}.$$

Assume that $V_1 \cap V_2 = \varnothing$. The join of G_1 and G_2 is defined as a fuzzy graph $G = G_1 + G_2 : (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on $G^* : (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E'$ where E' is the set of all edges joining vertices of V_1 with vertices of V_2, with

$$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u) \text{ for all } u \in V_1 \cup V_2$$

and

$$(\mu_1 + \mu_2)(uv) = \begin{cases} (\mu_1 \cup \mu_2)(uv), & uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(v), & uv \in E' \end{cases}.$$

By Radha and Kumaravel [8], the degree of an edge uv is defined

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = \sum_{\substack{uw\in E \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E \\ w\neq u}} \mu(wv) \tag{1.3}$$

2. Degree of an edge in Corona product

In this section, we give the definition of corona product operation and calculated degree of an edge of fuzzy graphs that are obtained by this operation.

The corona of two graphs is defined in [3] and there have been some results on the corona of two graphs [2]. The corona product of two graphs G and H; denoted by $G \circ H$; is the graph obtained by taking one copy of G of order n and n copies of H, and then joining by an edge the i-th vertex of G to every vertex in the i-th copy of H. The corona product is neither associative nor commutative. Let σ_i be a fuzzy subset of V_i and let μ_i be a fuzzy subset of E_i, $i = 1,2$. Using definition of join and union, define the fuzzy subset $\sigma_1 \circ \sigma_2$ of V and $\mu_1 \circ \mu_2$ of E as follows:

$$(\sigma_1 \circ \sigma_2)(u) = (\sigma_1 \cup \sigma_2 \cup \ldots \cup \sigma_2)(u) \quad (\text{union of } \sigma_1 \textit{ and } p_1 \text{ times } \sigma_2)\ \forall u \in V \tag{2.1}$$

$$(\mu_1 \circ \mu_2)(uv) = \begin{cases} (\mu_1 \cup \mu_2 \cup \ldots \cup \mu_2)(uv), (\text{union of } \mu_1 \textit{ and } p_1 \text{ times } \mu_2) & uv \in E - E' \\ \sigma_1(u) \wedge \sigma_2(v), & uv \in E' \end{cases} \tag{2.2}$$

Where E' is the set of all edges joining by an edge the the i-th vertex of G to every vertex in the i-th copy of H.

Theorem 2.1. Let $G = G_1 \circ G_2$. For any $uv \in E$,

1) If $uv \in E - E'$ then

$$d_G(uv) = \begin{cases} d_{G_1}(uv) + \sum_{w\in V_2} \sigma_1(u) \wedge \sigma_2(w) + \sum_{w\in V_2} \sigma_1(v) \wedge \sigma_2(w), & uv \in E_1 \\ d_{G_2}(uv) + \sum_{w\in V_1} \sigma_1(w) \wedge \sigma_2(u) + \sum_{w\in V_1} \sigma_1(w) \wedge \sigma_2(v), & uv \in E_2 \end{cases} \tag{2.3}$$

2) If $uv \in E'$ then

$$d_G(uv) = d_{G_1}(u) + d_{G_2}(v) + \sum_{\substack{uw\in E' \\ w\neq v}} \sigma_1(u) \wedge \sigma_2(w) \tag{2.4}$$

Proof: By (1.3), we have

$$d_G(\text{uv}) = \sum_{\substack{uw\in E-E' \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E-E' \\ w\neq u}} \mu(wv) + \sum_{\substack{uw\in E' \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E' \\ w\neq u}} \mu(wv) \tag{2.5}$$

Assume that $uv \in E - E'$ with $uv \in E_1$. Using (2.2) in (2.5) we get

$$d_G(uv) = \sum_{\substack{uw\in E_1 \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E_1 \\ w\neq u}} \mu(wv) + \sum_{\substack{uw\in E' \\ w\in V_2}} \sigma_1(u) \wedge \sigma_2(w) + \sum_{\substack{wv\in E' \\ w\in V_2}} \sigma_2(w) \wedge \sigma_1(v)$$

Using definition of (1.3), we obtain Theorem 1 (1) for $uv \in E_1$. Assume that $uv \in E_2$. In similar way we obtain Theorem 1 (1) for $uv \in E_2$.

Now, let $uv\in E'$ with $u\in V_1, v\in V_2$. Using (2.2) in equation (2.5) we see that

$$d_G(uv)=\sum_{w\in V_1}\mu(uw)+\sum_{w\in V_2}\mu(wv)+\sum_{\substack{uw\in E'\\ w\neq v}}\sigma_1(u)\wedge\sigma_2(w)+\sum_{\substack{wv\in E'\\ w\neq u}}\sigma_1(w)\wedge\sigma_2(v)$$

From definition of corona product, if $uv\in E', u\in V_1, v\in V_2$ and $w\neq u$ then $w\notin V_1$. As a conclusion, by (1.1) we obtain equation (2.4). Thus, we complete proof of the theorem. □

In the following theorems, we find the degree of uv in G in terms of those in G_k for $k=1,2$ in some particular cases.

Nagoorgani and Radha in [5] defined the relation $\sigma_1\geq\sigma_2$ means that $\sigma_1(u)\geq\sigma_2(v)$, for every $u\in V_1$ and for every $v\in V_2$, where σ_i is a fuzzy subset of $V_i, i=1,2.$

Theorem 2.2. Let $G=G_1\circ G_2$. For $\sigma_2\geq\sigma_1$ the following equalities holds:

1) If $uv\in E-E'$ then

$$d_G(uv)=\begin{cases} d_{G_1}(uv)+p_2\left(\sigma_1(u)+\sigma_1(v)\right), & uv\in E_1\\ d_{G_2}(uv)+2\sigma_1(w),\ w\in E_1\ and & uv\in E_2\end{cases}.$$

2) If $uv\in E'$ with $u\in V_1,\quad v\in V_2$ then

$$d_G(uv)=d_{G_1}(u)+d_{G_2}(v)+(p_2-1)\sigma_1(u)$$

Proof: We have $\sigma_2\geq\sigma_1$. Let any $uv\in E-E'$. From equation (2.3) for $uv\in E_1$ we have

$$d_G(uv)=d_{G_1}(uv)+\sum_{w\in V_2}\sigma_1(u)+\sum_{w\in V_2}\sigma_1(v).$$

Recall that $|V_i|=p_i, i=1,2.$ Hence, we have

$$d_G(uv)=d_{G_1}(uv)+p_2\left(\sigma_1(u)+\sigma_1(v)\right).$$

Now, for $uv\in E_2$, from equation (2.3) we have

$$d_G(uv)=d_{G_2}(uv)+\sum_{w\in V_1}\sigma_1(w)+\sum_{w\in V_1}\sigma_1(w)$$

Therefore we obtain Theorem 2.2 (1).
Using conditions of the Theorem 2.2 in equation (2.4), we get that

$$d_G(uv)=d_{G_1}(u)+d_{G_2}(v)+\sum_{\substack{w\in V_2\\ w\neq v}}\sigma_1(u)$$

Now, using equation (1.2) and $|V_i|=p_i, i=1,2$, we obtain that

$$d_G(uv)=d_{G_1}(u)+d_{G_2}(v)+(p_2-1)\sigma_1(u)$$ □

Theorem 2.3. Let $G=G_1\circ G_2$. For $\sigma_1\geq\sigma_2$ the following equalities holds:

1) If $uv\in E-E'$ then

$$d_G(uv) = \begin{cases} d_{G_1}(uv) + 2O(G_2), & uv \in E_1 \\ d_{G_2}(uv) + \sigma_2(u) + \sigma_2(v), & uv \in E_2 \end{cases}.$$

2) If $uv \in E'$ with $u \in V_1, \quad v \in V_2$ then

$$d_G(uv) = d_{G_1}(u) + d_{G_2}(v) + O(G_2) - \sigma_2(v)$$

Proof: We have $\sigma_1 \geq \sigma_2$. Let any $uv \in E - E'$. In similar a way, by equation (2.3) for $uv \in E_1$ we have

$$d_G(uv) = d_{G_1}(uv) + \sum_{w \in V_2} \sigma_2(w) + \sum_{w \in V_2} \sigma_2(w).$$

By equation (1.2), we get

$$d_G(uv) = d_{G_1}(uv) + 2O(G_2).$$

For $uv \in E_2$, from equation (2.3),

$$d_G(uv) = d_{G_2}(uv) + \sum_{w \in V_1} \sigma_2(u) + \sum_{w \in V_1} \sigma_2(v)$$

From definition of corona product, we obtain Theorem 2.3 (1). For any $uv \in E'$ with $u \in V_1, \quad v \in V_2$. Using (2.4), we get

$$d_G(uv) = d_{G_1}(u) + d_{G_2}(v) + \sum_{\substack{w \in V_2 \\ w \neq v}} \sigma_2(w)$$

By equation (1.2), we obtain Theorem 2.3 (2). □

Theorem 2.4. The corona product of two effective fuzzy graphs is an effective fuzzy graph.

Proof: Let $G_1(\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be effective fuzzy graphs. Then $\mu_1(u_1v_1) = \sigma_1(u_1) \wedge \sigma_1(v_1)$ for $uv \in E_1$ and $\mu_2(u_2v_2) = \sigma_2(u_2) \wedge \sigma_2(v_2)$ for $uv \in E_2$. Let $G = G_1 \circ G_2$. By (2.2), the fuzzy subset $\mu_1 \circ \mu_2$ of E is

$$(\mu_1 \circ \mu_2)(uv) = \begin{cases} \left((\sigma_1(u_1) \wedge \sigma_1(v_1)) \cup (\sigma_2(u_2) \wedge \sigma_2(v_2)) \cup \ldots \cup \mu_2 \right)(uv), uv \in E - E' \\ \sigma_1(u) \wedge \sigma_2(v), & uv \in E' \end{cases}$$

Thus, the proof of the theorem is completed. □

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