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Choosing the Appropriate E-Business Solution when doing E-Trade – An Application of Fuzzy Logic

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Abstract

Companies have to choose appropriate e-business software to offer their products and services online because such solutions have different features (i.e. flexibility or costs) whereby the aim is to fulfill the objectives and the requirements of the company.

This study deals with a selection problem which can be classified as a multi-attributive decision. This type of a problem often involves many vague and imprecise initial situations. In this paper, a possible fuzzy decision making model which is worked on by J.J. Buckley is examined. This method has proven itself as a valuable and applicable method for decision makers in real issues. In case of Analytical Hierarchy Process (AHP) model of Saaty, experts provide exact ratios which they personally judge as true. However, as this determination is a subjective approach, it may provide incomplete information (subjective method).

Due to the situation of unclear information, the use of fuzzy logic is crucial. So, the model of Saaty has been extended by Buckley to a case in which experts propose fuzzy ratios on which a good result is supported. This method can help companies to choose the most suitable solution for them in order to meet the challenges related e-business.

Keywords: Software solution, Selection problem, Multi-attributive decision, Analytical Hierarchy Process (AHP), Fuzzy AHP

1. Introduction

Companies try to offer their products and services online to customers quickly. At the same time they can use Internet for the procurement of relief supplies, which are necessary for the production of certain goods, i.e. within a few seconds an order can be placed. Thus, companies need the right infrastructure to offer their services online and that means they need competent, strong, efficient, and flexible e-business software that enables them to achieve the optimum for this challenge. The choice of an e-business software, which fulfills the objectives and requirements of a company, is of enormous importance. These software packages have different strengths. One may have very good marketing and sales tools and the other offers flexibility, expandability, and security tools. That is to say, it is important to know which software has which strengths and which weaknesses. Consequently, it is important for a growing company to use an e-business software where properties such as flexibility and expandability are abundantly present. Another issue which must be clarified in this context is that

which suppliers and thus which software should be considered. The treatment of this aspect is particularly important because there are many providers of e-business software.

An effective decision can be taken only if the large number of suppliers are reduced to a small number alternatives, thus to keep the chance of choosing alternatives, which completely fulfill the target of the company, as high as possible. Therefore, for the current purpose only, the following five products from reputable companies can be considered which are:

1. Intershop Enfinity Suite (Intershop)
2. Art Technology Group's Dynamo Commerce Suite (ATG)
3. Broadvision's One-to-One-Commerce (Broadvision)
4. IBM's Websphere Commerce Suite (IBM)
5. Microsoft's Commerce Server (Microsoft)

The aim of this study is to evaluate these products with the mathematical method fuzzy AHP, which is a known and worldwide proven method. Due to the available situation of vague information the use of fuzzy logic is crucial. In the section 'multi-attributive decision-making with the fuzzy logic' the importance of fuzzy logic is discussed. In order to see the advantages of fuzzy logic, the theoretical basics of fuzzy logic are outlined. Moreover, the method of Buckley (fuzzy AHP) is examined in every detail.

2. Overview of Relevant Properties of E-Business Solutions

The focus of this paper is an analysis of e-business software solutions from the economical aspect for a company. A company has three options when choosing the right solution:

1. It can buy a ready-made solution;
2. It is possible to rent a solution from a web hoster; or
3. It can create his own solution.

In this work only the first case will be considered. A detailed description or indication of all existing products on the market goes beyond the scope of the paper. Just the proven software solutions will be examined.

Software will be evaluated based on their certain properties. Choosing the right solution for the proposed business and its implementation are of enormous importance. Furthermore, all of the software solutions are considered precisely based on the following characteristics (ATG, Broadvision, Intersop, IBM, Microsoft, and Scheer):

1. Flexibility: Is this product upgradeable? What are the options to expand the product?

2. Integration: Are there other systems which can be integrated into the product? Are there lots of finished modules available? Are there many finished modules available?

3. Application: How is the application from the developer's perspective? For example, the shop care, whether it's too slow in the product range.

4. Development: Which type of tools are available? For example, templates and pipelets.

5. Database access: Is an extra database needed? How does it work with the access to the main database?

6. Costs: Costs relate to the purchase price, training costs, maintenance costs, and development costs for customers. One does not only need the software but also the required hardware. In general, the hardware is cheaper than the software. The importance of software comes from the fact that a stronger software is needed in order to meet the busyness of the system. That means to complete more transactions a more robust software is required. It was impossible to gain information on this feature for software. Therefore, costs will be estimated.

7. Sales: How does the process work when selling, i.e. the functionalities in order to sell products. Are these items placed in a shopping basket when purchasing products? Are variations of products ascertainable (i.e. different colors and sizes)? May the products be combined to groups or categories? Which promotional opportunities are available (discounts)? Are there several alternatives

available for the payment (direct debit, cash on delivery order and credit cards)? Which delivery options are available?

8. Marketing: How can marketing be carried out with the support of the system? May regular customers be notified, for example via e-mail about new products, discounts or special offers?

9. Order process: It is about the functionality of different payment methods and delivery methods. Are there separate billing and delivery addresses as functionality available? How does the shopping cart work?

10. Customer Service: Which support services does manufacturing company (such as Intershop) provide to one - developers, two - from developers to the shop owners (as Chibo in the case of Intershop) and three - from shop owners to end customer?

11. Report / Analysis: Which tools are available? What can be done with them?

12. Content Management: How is the maintenance of the products achieved? Is the content management system already integrated into the existing system or not?

13. Security: How is the security guaranteed? With which methods will it be achieved?

The processing of business transactions over the Internet will be possible by appropriate software. It can be used in the business areas from upstream warehouse ordering systems up to downstream after-sales solutions. The selection of the appropriate application is based on the criterion of whether they can be optimally integrated into existing processes.

With the help of the product spectrum of a company, one can evaluate the new value-added potential better. Digitized products, e.g. for music, software, and books are easier to sell or to advertise on the Internet. This possibility of quick delivery on the Internet does not exist for hardware products.

E-business solutions make it possible for companies to sell their products at lower prices due to little or no cost for production, packaging, and transport. So, a higher profit can be directly achieved. If necessary, the product will be sold at a lower sales price than the traditional way. Consequently, more products can be sold. Choosing the appropriate solution will facilitate the implementation phase of the business processes (Scheer, pp. 30-33). An effective solution covers the entire value chain. At the same time such software will offer the company long lasting advantages over its competitors through process optimizations and an increased communication to business partners and customers. The realization of these benefits requires an efficient e-business solution. Furthermore, a good solution will make it possible to build more efficient e-business systems. Accordingly, sales opportunities between companies or end customers through the internet are more facilitated for a company. It will allow the company to increase its profitability and it will simultaneously build solid and profitable relationships with its customers or partners.

Rentable relationships are of enormous importance for a company, since acquisition of new business partners and end customers/end users is much more expensive and time-consuming than the continuation of relations with existing partners. Therefore, lasting relationships are hugely significant for a company. So, such software should enhance customer satisfaction. At the same time, the relationship between company and customers should be made more effective. For example, a company can increase its sales figures through more exquisite presentation in the multimedia formats. This method of presentation is particularly suitable with products, which require explanation or when priority has to be given to emotional aspect in the product focus. The next aspect that a software solution should include is whether it provides a developer support. The most important question is whether it is quickly implemented.

The availability of an open and flexible architecture plays a major role and in particular this should be modular allowing for expansion as required. Whether it supports several languages is also important. In addition, optimization of the system should be supported, that is, the user should be able to measure, assess, and analyze the performance of the company.

The most important aspect is whether the e-business solution is available 24 hours a day. If the software model fails - even for a short period - it means the loss of customers and capital for a company.

Companies should also be in a position to engage and integrate business partners or suppliers in order to make cooperation and collaboration possible. This allows them to collect and apply all the information they may need, e.g. inventory (customer's warehouse stock), delivery times of suppliers, current employee presence, and access to customer information. If the supplier has access to sales forecasts of his customers, he is more likely to produce on time and plan the production process, e.g. in the respect of employment. At the same time, he can also try to keep the stock as low as possible. The company can try to find out by personalization, namely by highly targeted questionnaires or surveys for end users the customer's interests, opinions, and emotions, so that these can be stored in its database. Thus, the supplier has access to this valuable information and can learn more about the end customer. By obtaining a better understanding of the customer's interests, it is also able to cover their needs and interests better. The use of such a solution model should be able to optimize the logistics, from warehousing to distribution.

E-business strategy requires a technology that reduces costs and strengthens customer loyalty and makes supplier relationships more effective (Forrester Research). Products and services on the internet are the key weapons for a company to compete on the web.

3. Selecting the Right Supplier

Regarding to the B2C, software solution should be able to increase the sales. It should also cause lower costs due to the exclusion of the traditional intermediary by selling directly to end users. Thus, new forms of purchase and offers are applied, such as short-term offers with overcapacity or the formation of purchase communities. This solution aims to optimize the CRM and ties customers closely to company through a one to one marketing.

If a customer on the market evaluates e-business solutions and services, it is actually difficult to find the objective and accurate information. This is due to the fact that software suppliers have media and advertisers under their influence. Therefore, the purchasers of such goods need specialized staff who are able to assess objectively.

In Table 1 the approaches and difficulties in selecting the best choice among suppliers regarding to the software are shown. It is very obvious which tasks must be performed to find an effective solution. On the right side of the project tasks, associated questions for performing these tasks are located.

Projects change constantly. Changes have great importance in today's business world. The most important reason why companies should carefully evaluate the products on the market or try to find the right supplier is that the winner of today may fall very quickly lose its leading position due to the rapid improvements in e-business software field.

Table 1: Difficulties in choosing an e-business technology (Forrester Research)

Phase	Project Tasks	Problem-Sources
Select suppliers	• Make a research of the market • Restrict to a short list of suppliers • Negotiate conditions	• Which suppliers are crucial? • How can be the cost of testing justified? • How can the best deal be obtained?
Implement and integrate the first project	• Establishment of a project plan • Assignment of internal and external developers • Testing of the application and its integration	• What problems does the project have? • Do we have required skills? • How high are the costs of application and integration?
Extend on the entire company	• Train the project manager • Support them in their project planning • Lend required technical resources	• How can we convince the project team that this is the right product? • What have we learned about the project that can be shared?
Update (latest version)	• Consider improving benefits with costs • Set priorities for module changes	• Is the new version better or worse? • Which modules are of huge benefit?

4. Classical Multi-Attributive Decision Problem

After the description of software solutions, it is necessary to evaluate them with different methods. Looking at this issue, it is noticed that this is in fact a selection problem. Zangemeister (pp. 35) defines this matter as follows:

Definition 1 (Selection Problem)

The amount of available alternatives is to be arranged considering given constraints in terms of important objectives and relevant preferences of the decision makers.

The solution of the selection problem is particularly difficult if:

- many objectives are present
- there is vague information
- a clear decision criterion does not exist
- there are different objectives
- the preference structure[1] is different
- time dependencies are relevant.

A multi-criterion decision problem is concerned with the development of a better to worse order among the competing alternatives. This obtained order is nothing else than the complete solution to this problem. Such problems are also faced in everyday life frequently, e.g. when buying a property. Important features of a house may be defined as follows: Finance, year of construction, neighborhood, size, public transport connection, general condition, existing modern facilities, and space in front of and behind of the house.

Before fuzzy methods are described, it is necessary at this point to characterize the classical multi-attributive decision problem. Let U be the evaluation matrix:

$$
\begin{array}{c}
\quad C_1 \quad \ldots\ldots \quad C_i \quad \ldots\ldots \quad C_m \\
\begin{matrix} A_1 \\ \ldots \\ \ldots \\ A_j \\ \ldots \\ \ldots \\ A_n \end{matrix}
\begin{bmatrix}
a_{11} & \ldots\ldots & a_{1i} & \ldots\ldots & a_{1m} \\
\ldots & & & & \\
\ldots & & & & \\
a_{j1} & \ldots\ldots & a_{ji} & \ldots\ldots & a_{jm} \\
\ldots & & & & \\
\ldots & & & & \\
a_{n1} & \ldots\ldots & a_{ni} & \ldots\ldots & a_{nm}
\end{bmatrix}
= U \\
\quad w_1 \quad \ldots\ldots \quad w_i \quad \ldots\ldots \quad w_m
\end{array}
$$

The alternatives A_j, j = 1,..,n, criteria C^i, i = 1,..,m, and the weights which stand for relative importance of attributes w_i, i = 1,..,m are assumed to be given. At this point, it is supposed that all i and j indices of a_{ij} and w_i have values from the set {1,2,..,10}. In doing so, a_{ij} characterizes the level of compliance of the alternative j regarding to the criterion i. Let entries a_{ij} and w_i be given by experts. The assessment of any alternative j can be calculated by the sum of a_{ij} values over all the attributes i. The following formula can be used to make a decision on the best alternative.

$$U_j = \frac{\sum_{i=1}^{m} w_i a_{ji}}{\sum_{i=1}^{m} w_i} \equiv A_j ,$$

[1] This term includes summary expression of the relative meanings attributed to the managers's objectives and benefits of different target alternatives.

The best alternative is: $D^* = \{ A_k \mid U_k = \max_j U_j \}$.

U and U_j characterizes respectively the utility function and the benefits achieved by the alternative j. The best solution is the one with the greatest benefit. This method is subjective, as there may be cases in which an expert gives a unilateral or incomplete information. The concrete calculation in this case will not be done because it is very easy and not objective. The form of the available information, which is provided from the decision maker, is crucial. Elements a_{ij} of the evaluation matrix U must not be numeric, they can be expressed linguistically (low, medium, high) and fuzzy (vague).

5. Multi-Attributive Decision-Making With the Fuzzy Logic

In the article of Hsiao-Fan Wang fuzzy logic is applied to multi-attributive decision-making method. The problem of multi-attributive decision deals with the calculation and determination of the ranking of available alternatives. In this case, the best alternative will be determined by comparing with the others.

This problem includes a set of alternatives with different criteria. It is not easy to determine which alternatives should be considered. For this reason, the fuzzy logic is used with its fuzzy sets theory which gives the possibility to handle vague decision-making situations (e.g., inaccurate information provided by experts). Wang describes the two most widely used and applied fuzzy methods as following:

1. The fuzzy-weighted average method
2. Fuzzy AHP (The method investigated in this article)

5.1. Fuzzy Multi-Attributive Decision Problem

Analogous to the conventional case, in the case of fuzzy multi-attributive decision problem the matrix looks as:

$$\begin{array}{c} \\ A_1 \\ \dots \\ \dots \\ A_j \\ \dots \\ \dots \\ A_n \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & \dots\dots & C_i & \dots\dots & C_m \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \dots\dots & \tilde{a}_{1i} & \dots\dots & \tilde{a}_{1m} \\ \dots & & \dots & & \\ \dots & & \dots & & \\ \tilde{a}_{j1} & \dots\dots & \tilde{a}_{ji} & \dots\dots & \tilde{a}_{jm} \\ \dots & & \dots & & \\ \dots & & \dots & & \\ \tilde{a}_{n1} & \dots\dots & \tilde{a}_{ni} & \dots\dots & \tilde{a}_{nm} \end{bmatrix} \\ \begin{array}{ccccc} \tilde{w}_1 & \dots\dots & \tilde{w}_i & \dots\dots & \tilde{w}_m \end{array} \end{array} = \tilde{D}$$

The expert provides information on both the attributes and the alternatives, then a fuzzy weighted average method can be applied. This is why a possible decision will be found.

$$\mu_{\tilde{D}_j} = \min_i (\min(\mu_{\tilde{w}_i}, \mu_{\tilde{a}_{ji}}))$$

$$D^* = \{ A_k \mid \mu_{\tilde{D}_k} = \max_j \mu_{\tilde{D}_j} \}$$

Methods for determining possible decisions are based on the extension principle, while the optimal decision can be determined by the ranking method. This process can be carried out using several methods. The approach of Bonissone is one of these methods. The exact function of his method will not be dealt in details. He assumes that all the fuzzy or numerical data can be described by trapezoidal fuzzy numbers (such fuzzy numbers are illustrated in the following section).

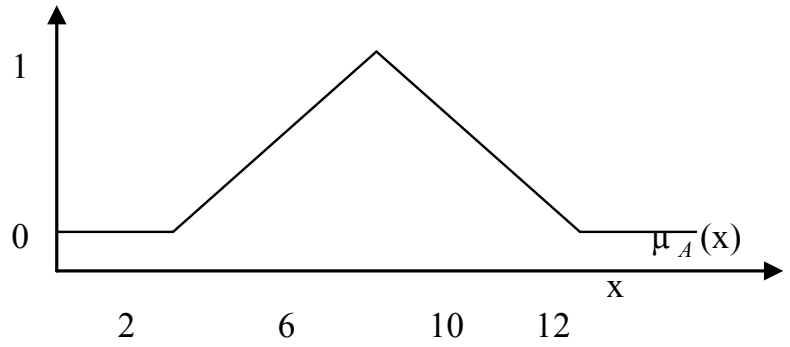
5.2. Fuzzy Numbers, Ratios and Their Significance

Definition 2 (Fuzzy set) (Rommelfanger, pp. 8)

Let X be a set of objects which have to be assessed regarding to a vague statement where A is a fuzzy set on X.

$A := \{ (x, \mu_A(x)) \mid x \in X \}$ mit $\mu_A : X \rightarrow [0, 1]$

The function μ(x) is called the membership function, characteristic function, or compatibility function. For example, the following figure is used to model the number 'close to six':



Note: The x-axis has been deliberately set below the value of y = 0 (μ (x) = 0), so it can clearly be seen where the membership function assumes the value zero.

The membership functions can be piecewise linear, i.e. they can take triangular, trapezoidal, or non-linear forms.

Definition 3 (Normalization) (Bernard, pp. 19):

A fuzzy set A is normalized if the following condition is satisfied:

$\sup_{x \in X} \mu_A(x) = 1,$

At least one element x ∈ X should have the membership degree one. In practice, normalized fuzzy-sets are used.

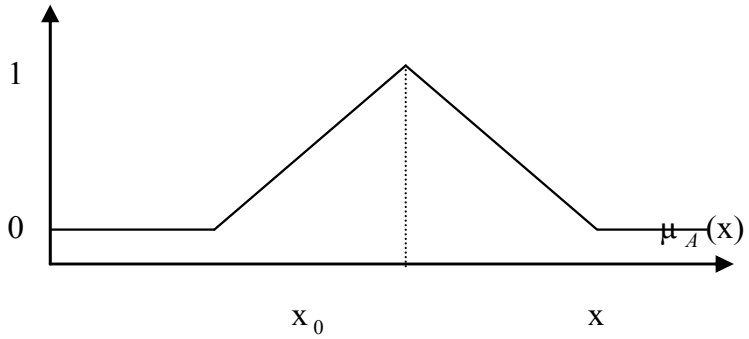
Definition 4 (Convexity) (Bernard, pp. 19):

A fuzzy set A is called convex, if:

$\mu(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2)) \; \forall \; x_1, x_2 \in X, \forall \; \lambda \in [0,1]$

Definition 5 (Fuzzy number) (Bernard, pp. 20 and Rommelfanger, pp.15)

A convex, normalized, and fuzzy set A is a (triangular) fuzzy number, if a real number x_0 exists with $\mu(x_0) = 1$ and μ (x) is piecewise continuous.



Note: This drawing is a possibility for a triangular fuzzy number which fulfills the definition

Definition 6 (Extension of the definition fuzzy number by **Dubois and Prade):**

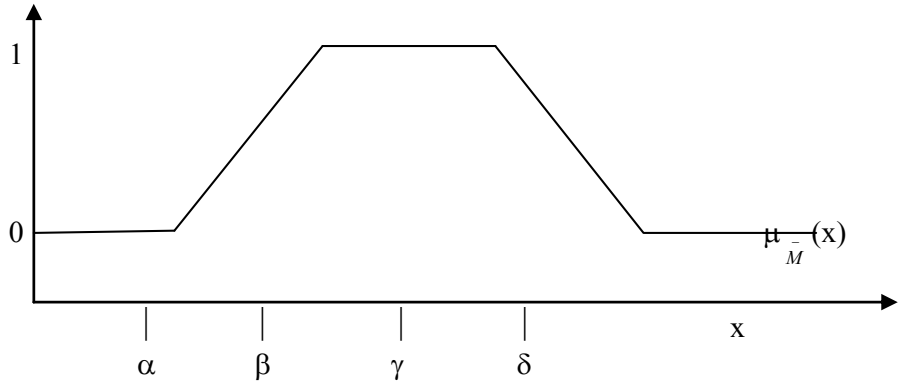
A real number $\bar{M}$ is described more precisely as any fuzzy subset of real numbers whose membership function $\mu_{\bar{M}}$ meets the following conditions where $\mu_{\bar{M}}$ is:

1) a continuous map from $\Re$ to the closed interval [0,1]
2) constant on $(-\infty,\alpha]$: $\mu_{\bar{M}}(x) = 0 \ \forall x \in (-\infty, \alpha]$
3) increases monotonically on the interval $[\alpha, \beta]$
4) constant on the interval $[\beta, \gamma]$: $\mu_{\bar{M}}(x) = 1 \ \forall x \in [\beta, \gamma]$
5) monotonically nonincreasing on the interval $[\gamma, \delta]$
6) constant on the interval $(\delta, +\infty)$: $\mu_{\bar{M}}(x) = 0 \ \forall x \in (\delta, +\infty)$

α, β, γ and δ are real numbers. One can also have cases, e.g. $\alpha = -\infty$, $\beta = \gamma$, $\gamma = \delta$ or $\delta = +\infty$.

Comments

- If $\alpha = \beta = \gamma = \delta$, then $\bar{M}$ is a normal real number.
- If $\alpha = \beta$ and $\gamma = \delta$, then $\bar{M}$ is a map of tolerance-interval $[\beta, \gamma]$ with the value one
- If $\beta = \gamma$, then $\bar{M}$ is a triangular fuzzy number.
- Two fuzzy numbers are equal if and only if they have the same membership function.



Fuzzy numbers enable a better articulation of the opinions of decision makers. For example, one can present a ratio of a_{ij} close to 4/1 instead of precisely 4/1 or a ratio between 5/1 and 7/1 instead 6/1. It is also clear that it is difficult to determine accurate values of a^{ij} when comparing two alternatives.

The statement 'A_i is more important than A_j' may mean a ratio of 6/1, 7/1, or 8/1 for a number of experts. Fuzzy numbers solve this problem by setting a relationship between 6/1 and 8/1. Thus, fuzzy numbers automatically take the uncertainty of the opinions of experts into account (Buckley, pp. 22).

J.J. Buckley defines a fuzzy number as a fuzzy subset of $\Re$ which can be described through $(\alpha/\beta, \gamma/\delta)$ where α, β, γ, and δ are real numbers where $\alpha \le \beta \le \gamma \le \delta$. The related membership function has the form which is shown in the last figure above. If A_i is concerning a criterion that is much more important than A_j, so the corresponding fuzzy-ratio has the following form:

$\bar{a}^{ij} = (\alpha^{ij}/\beta^{ij}, \gamma^{ij}/\delta^{ij})$ with $\alpha^{ij}, \beta^{ij}, \gamma^{ij}, \delta^{ij} \in \{1, 2, ..., 9\}$.

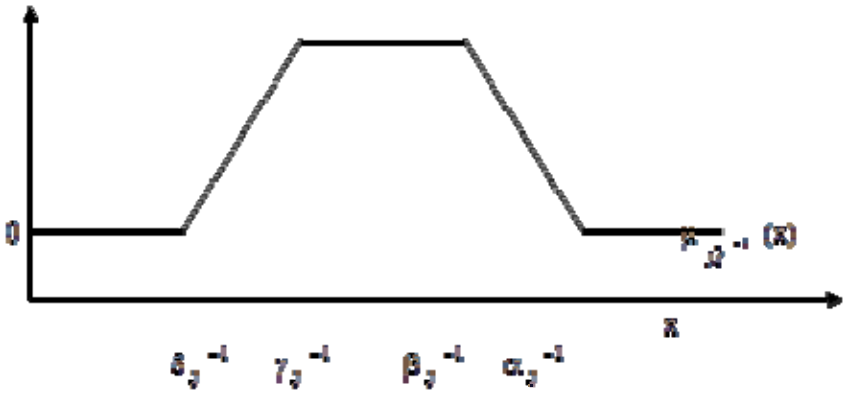
The reciprocal value of this is $\bar{a}_{ji}$ which means $(\bar{a}_{ij})^{-1} = (\delta_{ij}^{-1}/\gamma_{ij}^{-1}, \beta_{ij}^{-1}/\alpha_{ij}^{-1})$.

With respect to the membership function the reciprocal of a fuzzy number takes the values below:

$$\forall\, x \in \Re \setminus \{0\},\ \mu_{\bar{M}^{-1}}(x) = \sup_{z} \neq \{\mu_{\bar{M}}(z) \mid x = 1/z\}$$

$$= \sup_{z} \{\mu_{\bar{M}}(z) \mid z = 1/x\} = \mu_{\bar{M}}(1/x).$$

Accordingly, the membership function $\mu_{\bar{M}^{-1}}$ of reciprocal of a fuzzy number $\overline{M}$ is presented in the figure below:



Fuzzy-number (*4/5*, *5/6*) on the interval (4, 6) is triangular. Fuzzy-number (*3/3*, *3/5*) is defined as a monotonically decreasing (straight) line from (3,1) (i.e. (x, μ_M (x)) to (5, 0). At all other points x, the membership function takes the value 0. It is important to know the meaning of fuzzy numbers exactly in order to work better with them.

For example, **approximate *7/1*** can be described as a fuzzy-number by **(*6/7, 7/8*)**. On the other hand, an evaluation between ***5/1*** and ***7/1*** can be presented as a fuzzy-number by **(*5/5, 7/7*)** or **(*4/5, 7/8*)**. Depending on the expert, the fuzzy number (***4/4***, ***4/6***) can be interpreted as at least ***4/1*** (Buckley, pp. 234).

5.3. Fuzzifying the Method of Saaty

Before introducing this method, the evaluation scale of Saaty is presented below. For pairwise comparison, the 9-point-scale of Saaty will be used as follows:

Table 2: 9-point-scale of Saaty for pairwise comparisons (Zimmermann, pp. 58)

Weight Ratio	Statement
1	goal i is as important as j
3	goal i is less important than j
5	goal i is more important than j
7	goal i is much more important than j
9	goal i is extremely more important than j
2, 4, 6, 8	in between values

In the case of Analytical Hierarchy Process (AHP) method which is developed by Saaty, the decision making expert provides accurate ratios a_{ij} by pairwise comparison between alternatives A_1, A_2,..., A_n for each criterion in a hierarchy and also in between the criteria. The numbers which are used for these ratios belong to the set {*1,2 ,..., 9*}.

If a decision maker considers alternative A_1 as much more important than A_2, then a_{12} can take values *3* to *1* which means *3/1*, *5/1*, or *7/1*. For certain experts these ratios a_{ij} state the strength in

which alternative A_i dominates the alternative A_j. Suppose $a_{12} = 7$, so in a reciprocal matrix $a_{21} = 1/7$ and $a_{ii} = 1$ for all *i*. Then *A* is called a positive reciprocal matrix. In order to determine the ranking of the alternatives from the best to the worst, the weights w_i ($w_i > 0$, $w_1 + w_2 + ... + w_n = 1$) must be defined for alternatives which can be determined with the method of Saaty. This ranking is done using pairwise comparison matrices concerning each criterion and using the pairwise comparison matrix *A* for each criteria. In fact, experts can not set accurate values by comparing the alternatives for each criterion or by comparison of criteria. This is justified by the fact that a decision maker considers for example A_i as more important than A_j. The corresponding ratios for this statement can be close to *3/1*, between *2/1* and *4/1*, or no more than *5/1*.

Values a_{ij} may be fuzzy-numbers. The fuzzy number, which has a triangular form, has been defined previously. In this section, this definition will be extended to a more general basis which also meets the recent case of $\beta = \gamma$ (Buckley, pp. 233-234) in special cases.

5.3.1. Determination of Fuzzy Weights and their Role in Setting the Ranking

To determine the weights w_i, the geometric mean method can easily be extended to the case of fuzzy positive reciprocal matrices $\bar{A}$. Let a positive reciprocal matrix $A = [a_{ij}]$ be given.

First of all, the geometric mean of each line can be calculated as:

$$r_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}$$

Afterwards, $w_i = r_i / (r_1 + ... + r_n)$ are determined.

There are other methods by which the weights of a fuzzy positive reciprocal matrix can be calculated. The article of J.J. Buckley deals with a method which can easily be extended to the case of fuzzy positive reciprocal matrices and has a number of desired properties. This is called the logarithmic least squares method which converges to the geometric mean method. The functionality or application of this method will be investigated by an example through concrete calculations.

Consequently, the weights for the matrix $\bar{A} = [\bar{a}_{ij}]$ are obtained as follows:

$$\bar{r}_i = (\bar{a}_{i1} \otimes ..... \otimes \bar{a}_{in})^{1/n} \text{ and } \bar{w}_i = \bar{r}_i \otimes (\bar{r}_1 \oplus ..... \oplus \bar{r}_n)^{-1}$$

With this formula only the points are determined directly, in which the corresponding membership function μ_i of $\bar{w}_i$ takes values zero or one. Therefore, with this result a statement about the behavior of the membership function between (*0, 1*) can not be made.

For these values, J.J. Buckley proposes the following formula:

$$f_i(y) = [\prod_{j=1}^{n} ((\beta_{ij} - \alpha_{ij})y + \alpha_{ij})]^{1/n}, 0 \le y \le 1$$

$$g_i(y) = [\prod_{j=1}^{n} ((\beta_{ij} - \alpha_{ij})y + \alpha_{ij})]^{1/n}, 0 \le y \le 1$$

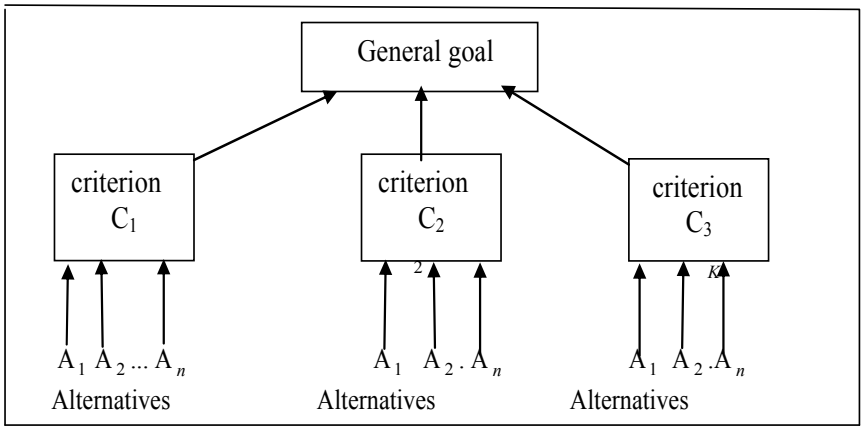
Suppose $\alpha_i = [\prod_{j=1}^{n} \alpha_{ij}]^{1/n}$ and $\alpha = [\sum_{i=1}^{n} \alpha_i]$, β_i and β, γ_i and γ, and δ_i and δ are analogously defined.

Finally, the following values are obtained (Buckley, pp. 237):

$$f(y) \sum_{i=1}^{n} f_i(y), \qquad g(y) = \sum_{i=1}^{n} g_i(y)$$

Consequently, better insights can be achieved with this method which can be illustrated by the following table:

Table 3: Hierarchical structure (Buckley, pp. 238)



The method addressed above has the goal of determining the fuzzy weights for a given positive reciprocal matrix $\bar{A}$. It requires a special hierarchy to calculate the final fuzzy weights and the final ranking of alternatives. Thus, by pairwise comparison of alternatives a fuzzy positive reciprocal matrix $\bar{A}_k$ with respect to a criterion C_k is obtained. In addition, a fuzzy positive reciprocal matrix $\bar{E}$ is obtained by pairwise comparison of criteria. Consequently, the fuzzy weights are determined as $\bar{w}_k = (\bar{w}_{1k},...,\bar{w}_{nk})$ for matrix $\bar{A}_k$ and as $\bar{e} = (\bar{e}_1,....,\bar{e}_k)$ for matrix $\bar{E}$.

The final weight for the alternative A can be calculated by using the following formula:

$$\bar{f}_j = (\bar{w}_{j1} \otimes \bar{e}_1) \oplus .....\oplus (\bar{w}_{jk} \otimes \bar{e}_k)$$

Membership functions $\bar{f}_j$ can easily be calculated from the membership functions $\bar{w}_j$ and $\bar{e}_j$. Moreover, every $\bar{f}_j$ can be multiplied by an appropriate constant value so that the support, $\bar{w}_i$ ($\mu_i(x) > 0$), of $\bar{f}_j$ is located on the interval [0, 1].

Definition 7 (Buckley, pp. 239)

Alternative A_i dominates alternative A_j, $A_i > A_j$, if and only if $\bar{f}_i > \bar{f}_j$. If A_i does not dominate alternative A_j and if A_j does not dominate A_i, then $A_i \approx A_j$.

Sets H^1, H^2,...., H_d are defined as: H^1 contains all alternatives A_i, which are not dominated by others. However, H^2 includes all alternatives $A_i \in \{A^1, A^2,...,A_n\} \setminus \{H^1\}$ which are not dominated by others. In the same way, this process is continued for other H_3, H^4,... etc. sets until last set H_d is obtained. If $H^1 \neq \varnothing$ and H^1 does not contain all possible alternatives, then H^2 is not empty. Similarly, for sets H_3, H^4,..etc. the same procedure is followed. That is to say, the alternatives in the set H_i are located on the same level in the ranking. Alternatives in H^1 have the highest positions and those in H^2 belong to the next-highest status, etc. The final ranking of alternatives will be obtained by determining the sets H^1, H^2,...., H_d.

5.3.2. Application on Concrete Example

In order to understand the steps of calculation, an example where three types of chemicals are classified from most harmful to least harmful by a government agency is given by Buckley (pp. 243-244). The agency defines three different criteria (aquatic life, agriculture, and timber). A group of

experts are employed by the agency who deliver the fuzzy reciprocal matrices A_{ik} for each criterion C_i. Furthermore, this agency collects data on pairwise comparisons of criteria in order to get the fuzzy positive reciprocal matrices E_i. It is important that each fuzzy positive reciprocal matrix is consistent. A reciprocal matrix using the evaluation scale of Saaty (Table 2) takes the following form for three alternatives A_1, A_2 and A_3 evaluated for each criterion C_1, C_2 and C_3. So, a cell will be illustrated by trapezoidal fuzzy number $\bar{a}_{ij}$. Such a fuzzy number has the form:

$$\bar{a}_{ij} := (a_{ij} / b_{ij,} c_{ij} / d_{ij}).$$

For criterion C_i the general matrix has following form:

		A_1	A_2	A_3
C_i	A_1	$(a_{11} / b_{11,} c_{11} / d_{11})$	$(a_{12} / b_{12,} c_{12} / d_{12})$	$(a_{13} / b_{13,} c_{13} / d_{13})$
	A_2	$a_{21} / b_{21,} c_{21} / d_{21})$	$(a_{22} / b_{22,} c_{22} / d_{22})$	$(a_{23} / b_{23,} c_{23} / d_{23})$
	A_3	$(a_{31} / b_{31,} c_{31} / d_{31})$	$(a_{32} / b_{32,} c_{32} / d_{32})$	$(a_{33} / b_{33,} c_{33} / d_{33})$

For pairwise comparisons of the criteria the general matrix *E* has the following form:

		C_1	C_2	C_3
E	C_1	$(a_{11} / b_{11,} c_{11} / d_{11})$	$(a_{12} / b_{12,} c_{12} / d_{12})$	$(a_{13} / b_{13,} c_{13} / d_{13})$
	C_2	$(a_{21} / b_{21,} c_{21} / d_{21})$	$(a_{22} / b_{22,} c_{22} / d_{22})$	$(a_{23} / b_{23,} c_{23} / d_{23})$
	C_3	$(a_{31} / b_{31,} c_{31} / d_{31})$	$(a_{32} / b_{32,} c_{32} / d_{32})$	$(a_{33} / b_{33,} c_{33} / d_{33})$

Such an example can be applied in the same way to e-trade software solutions where experts supply fuzzy reciprocal matrices A_{ik} for each criterion C_i and fuzzy positive reciprocal matrices E_i.

In the 'Overview of Relevant Properties of E-Business Solutions' section five solutions were preferred among other software solutions (such as Intershop Enfinity Suite and Art Technology Group's Dynamo Commerce Suite) which stand for alternatives $A_1,...,A_5$ with 13 criteria (e.g. flexibility, integration, and application) where they are represented through $C_1,... C_{13}$. After experts give their opinion for matrices A_{ik} and E_i, the method of Buckley may be applied in order to get a good result for the evaluation of e-business solutions with consideration of multi-attributive decisions. In order to gain practical insight about how fuzzy AHP method functions work, the above mentioned example about chemicals can be used.

6. Observations and Conclusions

The aim of this study is that the data obtained about the features of e-business software solutions needs to be investigated by effective mathematical methods (for example by weighting the criteria or features of software). This enables making a decision on right choice for a company, because experts provide inaccurate information on both the attributes and the alternatives.

The Saaty's method (AHP) is subjective which is the reason why it has not been presented in details, because otherwise it will go beyond the scope of this work. But, this method has been "fuzzified" with the tools of fuzzy logic so that it is illustrated comprehensively.

The quality of data is dependent on the opinion of experts. Due to vague and incomplete data on products, the method of fuzzy logic (theory of vague sets) is an appropriate tool to apply with which a real decision making problem can be modeled.

The data and their interdependencies can be included into the model by means of fuzzy logic (Fuzzy AHP). Another advantage of the use of fuzzy logic is that it is less likely to represent the existing problem through a wrong model. The application of fuzzy theory to the AHP method gives sound results.

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